

Mathematics: understanding the score

Messages from inspection evidence

This report is based principally on evidence from inspections of mathematics between April 2005 and December 2007 in 192 maintained schools in England. Part A focuses on the inspection findings in the context of rising standards over the last decade in national tests and examinations. Part B discusses the issues underlying the rises in results and describes the essential components of effective mathematics teaching. Illustrative examples and brief commentaries are provided. The report's findings contributed to the review of mathematics led by Sir Peter Williams and published in June 2008.

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Executive summary

This report is based on evidence from inspections of mathematics between April 2005 and December 2007 in 192 maintained schools in England. It also draws on evidence from whole-school inspections from September 2005 to July 2007; from visits relating to the evaluation of the National Strategies during the same period; from Ofsted's previous reports; and from discussions with teachers and others.

In the 84 primary and 108 secondary schools in the survey sample, the effectiveness of work in mathematics was outstanding in 11%, good in 44% and satisfactory in 40%. Of the nine schools where the quality was inadequate, six were secondary schools. The recent increase in the proportion of primary schools where provision for mathematics was judged to be outstanding is encouraging. Many secondary schools were aware that their work in mathematics was an area of relative weakness and were trying to improve, often in challenging circumstances that included staffing difficulties.

The last decade has seen significant rises in standards in mathematics for pupils of all ages, as evidenced by data from national tests and public examinations. Recently, however, the rate of improvement has slowed in Key Stage 2 and stalled in Key Stage 1. In part, this is because pupils who begin formal education with relatively weak mathematical skills need to make more progress than many of their peers if they are to reach the expected levels by the end of the key stage. Many primary teachers require deeper subject knowledge if they are to help these pupils to make the necessary gains in order to close the gap and move forward confidently.

Key Stage 3 test results are improving and a greater percentage of pupils reach the vital threshold of grade C at GCSE level, but this does not tell the whole story. Based on the gains made at Key Stage 3, more pupils than at present should be reaching the higher GCSE grades. Evidence suggests that strategies to improve test and examination performance, including 'booster' lessons, revision classes and extensive intervention, coupled with a heavy emphasis on 'teaching to the test', succeed in preparing pupils to gain the qualifications but are not equipping them well enough mathematically for their futures. It is of vital importance to shift from a narrow emphasis on disparate skills towards a focus on pupils' mathematical understanding. Teachers need encouragement to invest in such approaches to teaching.

At AS and A level, pass rates have continued to rise and the numbers of entries have recovered strongly from the sharp drop following the introduction of Curriculum 2000. The Government's target of 56,000 entries by 2014, roughly 10% of the cohort, seems to be within reach. However, mathematics continues to attract predominantly the highest attaining pupils, and many more boys than girls. Widening its appeal has not yet been very successful.

Part B of the report discusses the issues in mathematics and barriers to improving learning, but also describes characteristics of good and outstanding practice. A shared philosophy about effective learning in mathematics often underpins the work of the best primary schools and secondary departments.

The fundamental issue for teachers is how better to develop pupils' mathematical understanding. Too often, pupils are expected to remember methods, rules and facts without grasping the underpinning concepts, making connections with earlier learning and other topics, and making sense of the mathematics so that they can use it independently. The nature of teaching and assessment, as well as the interpretation of the mathematics curriculum, often combine to leave pupils ill equipped to use and apply mathematics. Pupils rarely investigate open-ended problems which might offer them opportunities to choose which approach to adopt or to reason and generalise. Most lessons do not emphasise mathematical talk enough; as a result, pupils struggle to express and develop their thinking.

Assessment has a vital part to play in building pupils' understanding of mathematics but it remains an area of weakness, particularly in secondary schools. This is not just about lesson objectives, questioning and marking, but about seeking and acting on clues from pupils' responses and their written work, noticing early errors and the sticking points that hold back learning. Teachers need to see the learning from each pupil's viewpoint and then use activities that progressively challenge their thinking.

The essential ingredients of effective mathematics teaching are subject knowledge and understanding of the ways in which pupils learn mathematics – drawn together in the report as 'subject expertise' – together with experience of using these in the classroom. The quality of teachers' subject expertise is uneven, varying largely, but not exclusively, by phase. In short, secondary teachers see themselves as teaching mathematics; primary teachers see themselves as teaching pupils. The fundamental areas for improvement, therefore, are the subject knowledge of primary and non-specialist teachers and the pedagogical skills of secondary teachers.

Pupils have the last word in the report: their views about learning mathematics, their understanding and enjoyment. During the survey visits, they confirmed the narrow nature of much of the teaching but they also showed how much difference a teacher's enthusiasm can make.

Key findings

- Results of national test at Key Stages 2 and 3 and GCSE examinations have shown an upward trend for several years. Outcomes in the Foundation Stage and Key Stage 1 have remained steady. Results continue to rise at AS and A level.
- Taking into account their starting points, pupils' achievement is at least satisfactory. Schools have used a range of intervention and other strategies to boost performance in tests and examinations, but a rising proportion of pupils do not sustain the gains they made at Key Stage 3 through to GCSE level.
- Teaching was good or better in just over half the lessons seen and satisfactory in around two in five. It was better in primary than secondary schools, mainly because of primary teachers' attention to the needs of individual pupils. Many secondary schools have difficulty in recruiting suitably qualified staff, particularly subject leaders. Too much teaching concentrates on the acquisition of sets of disparate skills needed to pass examinations.
- The best teaching in both phases was enthusiastic, knowledgeable and focused clearly on developing pupils' understanding of important concepts. Good assessment throughout the lesson enabled the teacher to see how pupils were thinking and to adjust teaching and learning strategies accordingly. By developing pupils' mathematical independence, teachers also equipped them for success in examinations and beyond.
- Pupils wanted to do well in mathematics. They knew it was important, but were rarely excited by it, were generally not confident when faced with unusual or new problems and struggled to express their reasoning. Their recall of knowledge and techniques was stronger than their understanding.
- Despite recent initiatives, assessment for learning continues to be relatively weak. Most teachers did not exploit fully its potential for checking on and promoting pupils' understanding, often because of shortcomings in their subject knowledge or pedagogic skills. Too few teachers moved around the class to check for pupils who were stuck, had made slips, or who found the work easy.
- The content of the mathematics curriculum in most of the schools surveyed was age-appropriate. However, the majority of pupils had too few opportunities to use and apply mathematics, to make connections across different areas of the subject, to extend their reasoning or to use information and communication technology (ICT). Higher-attaining pupils were not always challenged enough in lessons. Links with other subjects were insufficient.
- Schemes of work in secondary schools were frequently poor, and were inadequate to support recently qualified and non-specialist teachers.
- The quality of leadership and management of mathematics was good or better in 71% of the primary schools and 51% of the secondary schools visited, although it has improved in the latter in the last two years. Schools' use of assessment data to identify pupils who are in danger of not meeting their targets has improved.
- In the more effective schools, collaboration between staff supported their professional development but, generally, opportunities for teachers to improve their subject knowledge and subject-specific pedagogy were infrequent.

Recommendations

The Department for Children, Schools and Families and the National Strategies should:

- explore strategies through which the subject expertise (knowledge of mathematics and of the ways pupils learn the subject) of all teachers of mathematics can be developed and lead to recognition and reward
- build on the recommendation from the Williams Review of mathematics teaching, by enhancing the role of subject leader for mathematics in primary schools so that teachers aspire to it and commit themselves to increasing the depth of subject knowledge that effective leadership demands¹
- introduce a range of incentives to support secondary schools in appointing and developing effective subject leaders for mathematics departments
- provide guidance for schools on enhancing subject expertise in mathematics
- devise guidance for teachers on the effective use of mathematics-specific pedagogy to aid the development of pupils' understanding
- reintroduce separate reporting of pupils' attainment in 'using and applying mathematics' as part of statutory teacher assessments at the end of each key stage; this would reflect the raised profile given to key concepts and processes in the new secondary National Curriculum.

The Qualifications and Curriculum Authority should:

- ensure current and future developments in external assessment place increased emphasis on pupils' understanding of mathematics and readiness for the next stage in their education, and avoid forms of assessment that fragment the mathematics curriculum.

Training providers and the Training and Development Agency for Schools should:

- ensure initial teacher education courses for all teachers of mathematics include relevant enhancement of subject knowledge and key mathematical concepts.

¹ *Independent review of mathematics teaching in early years settings and primary schools*, 2008; <http://www.standards.dcsf.gov.uk/primary/mathematicsreview>

The National Centre for Excellence in the Teaching of Mathematics should:

- further develop diagnostic tools for teachers' self-assessment of subject knowledge and provide information about relevant courses, distance-learning modules and regional support activities, making sure gaps in provision are tackled.
- collaborate with the National Strategies and other providers to ensure all teachers of mathematics have ready access to training on subject-specific pedagogy
- work with local authorities, and other groups such as subject associations, to improve opportunities for networking to share good practice locally and to promote developmental work with harder-to-reach staff.

Schools should:

- improve subject leaders' expertise so that they are well placed to lead improvements in the teaching and learning of mathematics and the curriculum
- encourage teachers to focus more on developing pupils' understanding and on checking it throughout lessons
- ensure pupils have a wide range of opportunities to use and apply mathematics, underpinned by thorough assessment, recording and reporting
- provide well targeted professional development in mathematics, particularly to improve teachers' subject-specific pedagogy and the subject knowledge of non-specialist teachers of mathematics
- identify and tackle underlying weaknesses in teaching that lie at the source of pupils' gaps in knowledge or difficulties in learning mathematics, thereby reducing reliance on short-term intervention strategies
- gather and take into account pupils' views on learning mathematics.

Primary schools should also:

- provide greater depth and challenge in lessons for the higher-attaining pupils.

Secondary schools should also:

- make use of flexibilities in pay and incentives to help mathematics departments overcome their distinctive challenges and support their development
- enhance schemes of work to include guidance on teaching approaches and activities that promote pupils' understanding and build on their prior learning
- improve pupils' use of ICT as a tool for learning mathematics.

Part A: Mathematics in primary and secondary schools

Achievement and standards

Pupils' performance in tests and examinations

1. Teachers' assessments show that standards in mathematical development in the Foundation Stage and in mathematics at Key Stage 1 have remained steady in recent years. Children in the Foundation Stage are best at counting and recognising shapes; they are not so good at calculating or describing position. At Key Stage 1, pupils extend their knowledge of shapes and numbers, counting, adding and subtracting, but are less confident about solving problems. Early multiplication and division also cause some difficulty.
2. In Key Stages 2, 3 and 4, results of national tests and examinations in mathematics have shown an upward trend for several years, although Key Stage 3 results dipped slightly in 2007, following a relatively large rise in 2006. As pupils move through primary and secondary school, they learn more about all areas of mathematics. For example, starting with whole numbers, they move on to decimals and fractions, positive and negative numbers, very large and very small numbers, and eventually on to rational and irrational numbers such as pi (π) and $\sqrt{2}$. Older pupils are increasingly competent at carrying out taught methods, such as solving equations or calculating the volumes of solid shapes. This stands them in good stead when they sit tests and examinations. They find it much more difficult, however, to use the skills they have learnt to solve more unusual problems and to identify connections between different skills and topics.
3. Table 1 shows the proportion of pupils reaching the expected attainment thresholds for each key stage in 2007 compared to 2001 and 2004. It also shows the proportions attaining or exceeding the higher Level 5 at Key Stage 2, Level 6 at Key Stage 3 and grade B at GCSE. More pupils than in the past are making two levels of progress during Key Stage 3, contributing to the increased percentages reaching Levels 5 and 6 by age 14. Even so, the Key Stage 2 and 3 figures of 77% and 76% reaching Levels 4 and 5 respectively still fall well short of the Government's targets of 85% at each key stage.

Table 1: Pupils reaching the expected attainment thresholds in mathematics for each key stage in 2001, 2004 and 2007

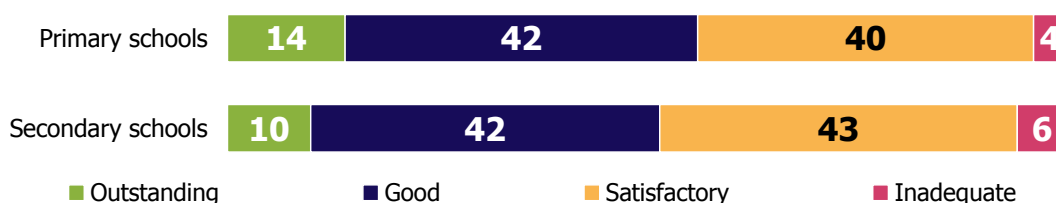
	Percentage of pupils achieving selected threshold indicators	2001	2004	2007	Government target (and target date)
Foundation Stage	Within the Early Learning Goals	n/a	n/a	66	
Key Stage 1	Level 2+	91	90	90	
Key Stage 2	Level 4+	71	74	77	85 (2006)
	Level 5+	25	31	33	
Key Stage 3	Level 5+	66	73	76	85 (2007)
	Level 6+	43	52	56	
Key Stage 4 (GCSE)	Grade C+	51	53	57	
	Grade B+	30	32	34	

4. The improvements made in Key Stage 3, however, are not built on sufficiently during Key Stage 4. Indeed, pupils' progress during Key Stage 4 has declined over the past few years. In 2007, 79% of pupils who had reached Level 6 at Key Stage 3 went on to pass GCSE at grade C or higher, and 26% did so from Level 5. These proportions are much lower than the corresponding figures for English and science. For mathematics in 2000, the figures were around 90% and 40% respectively. The question is whether the depth of understanding required to reach Level 5 or 6 in tests at the end of Key Stage 3 is sufficient to prepare pupils for their future study of mathematics. Inspection evidence throws light on this and other factors affecting progress during Key Stages 3 and 4.
5. Participation in AS and A-level mathematics has increased markedly since the changes to specifications for courses starting in September 2004. This is making up the ground lost following the introduction of Curriculum 2000. A-level entries among 16- to 18-year-olds exceeded 53,000 in 2007, which is nearing the figure in 2001, having fallen sharply to below 45,000 in 2002 and 2003. The Government's target of 56,000 entries by 2014 now appears to be within reach. Nevertheless, entries are still considerably lower than the peak of 63,000 in 1990.
6. Mathematics was boys' most popular subject at A level in 2007 and many more boys than girls studied it. Taking into account their GCSE starting points, the achievement of boys and girls is broadly equal. However, students from some minority ethnic groups and those eligible for free school meals are under represented at A level. Pass rates at AS level have improved significantly from 69% in 2001 to 81% in 2007, but remain lower than in most other subjects. Although the highest GCSE grades are not specified as prerequisites for advanced level study of mathematics, many students who attained grades C or B at GCSE struggle to gain a pass grade at AS level, and

many do not subsequently proceed to A level. This again raises questions about the quality of students' earlier learning in terms of preparation for further study.

7. The pass rate of those taking A-level mathematics has risen to 97%, and more students than ever are gaining high grades. This trend, which has persisted for some time, accelerated with the introduction of new courses in 2004 which reduced the amount of content to be studied.² The proportion awarded grades A or B reached 65% in 2007, around 20 percentage points higher than in 2001. Most of these students had achieved grades A or A* at GCSE.
8. An important post-16 success story has been the growth of further mathematics since the launch of the Further Mathematics Network in response to the Smith Report.^{3,4} A national network of 46 further mathematics centres provides access to all GCE specifications in further mathematics. The network aims to make AS and A-level further mathematics qualifications available to every student who would benefit and more than 1,200 schools and colleges are now registered. Over the last two years, further mathematics entries have increased by 44% at AS and 40% at A level. The slight fall in results in 2007 reflects the widening range of students' prior attainment on entry to the courses.
9. Inspectors judge how well pupils have achieved in mathematics when their varied starting points are taken into account. Achievement was judged to be good or better in just over half of the schools visited during the period of this survey (Figure 1).

Figure 1: Achievement in mathematics in the schools surveyed (percentages of schools)



Figures should be treated with caution due to sample sizes. Percentages are rounded and do not always add exactly to 100.

10. Although the proportions were broadly similar in primary and secondary schools, there was a clear difference between the phases in how well pupils learnt mathematics on a day-to-day basis. Secondary pupils made good progress in just under half the lessons observed. Nationally, this needs to be improved if all pupils' life chances are to be enhanced.

² A report on the effect of this change, *Evaluation of participation in A-level mathematics: Interim report, Autumn 2005*, (QCA/J2326), QCA Research Faculty, 2006, found widespread agreement among teachers that the changes made A-level mathematics easier, but disagreement about whether this was a good thing. <http://ofqual.gov.uk/987.aspx?q=Evaluation+of+participation+in+A-level+mathematics&submit.x=20&submit.y=16>

³ www.fmnetwork.org.uk.

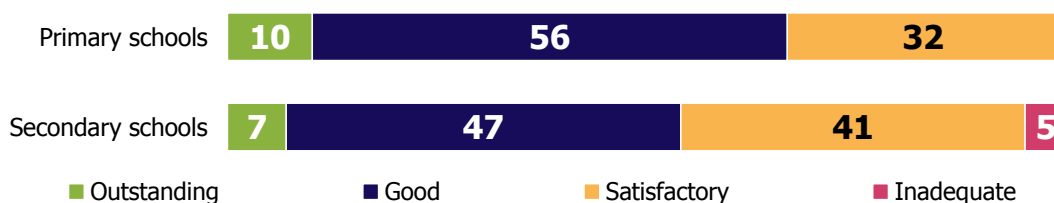
⁴ *Making mathematics count: The report of Professor Adrian Smith's inquiry into post-14 mathematics Education*, The Stationery Office (937764), 2004.

Teaching and learning

The quality of teaching and learning in mathematics

11. The quality of teaching and learning in mathematics was good and sometimes outstanding in 60% of the 192 schools visited. It was substantially better in primary than in secondary schools (Figure 2).

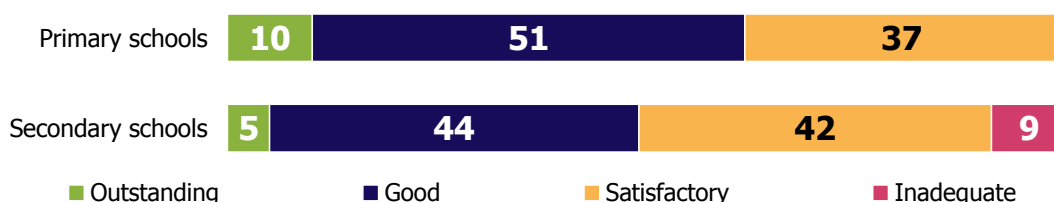
Figure 2: Quality of teaching and learning in mathematics in the schools surveyed (percentages of schools)



Figures should be treated with caution due to sample sizes. Percentages are rounded and do not always add exactly to 100.

12. The proportion of good lessons seen in primary schools was highest in Key Stage 1 and the Foundation Stage and lowest in Years 3 and 4. Pupils in mixed-age classes were less likely to receive good teaching than those in single-age classes. In secondary schools, the quality of teaching was highest in the sixth form and there was little difference overall between Key Stages 3 and 4. Pupils' progress was inadequate in nearly 10% of secondary mathematics lessons (Figure 3).

Figure 3: Progress in mathematics lessons in the schools surveyed (percentages of lessons)



Figures should be treated with caution due to sample sizes. Percentages are rounded and do not always add exactly to 100.

What does good or outstanding teaching look like?

13. The best teaching was rooted in developing pupils' understanding of key concepts. It was inclusive in terms of ensuring that all pupils made substantial progress, no matter what their starting points. In the outstanding lessons, the teachers had high expectations of pupils' enjoyment and achievement. They made conscious efforts to foster a spirit of enquiry, developing pupils' reasoning skills through approaches that saw problem-solving and investigation as integral to learning mathematics. They checked that everyone was challenged to think hard and they adapted how they were teaching to achieve this. As a result, their classrooms were vibrant places of learning.

Prime practice: teaching for understanding	This Year 9 lesson on the volume of cylinders enabled pupils to improve their estimation skills greatly and to understand the formula to find volume.
<p>First, each pupil wrote an estimate for the volume of a tea candle that was on their desk. These estimates were generally far below the actual volume. The teacher then used a demonstration on the interactive whiteboard, checking very carefully that everyone could interpret the two-dimensional representation of circular layers gradually building up and could explain how the formula for the volume linked to their previous knowledge. Pupils worked in pairs with everyday objects that were well chosen for their dimensions, making measurements and calculating volumes. In doing this, they became much clearer about the size of a cubic centimetre, estimating how many would fit into an object. At the end of this very well organised lesson, pupils were much more accurate in estimating the volume of the tea candle by eye and most were very surprised that it was many more cubic centimetres than they had initially estimated.</p> <p>The planning of the lesson had skilfully brought about a mismatch between the pupils' initial estimates and the actual volume. This added greatly to their learning as their surprise deepened their thinking and led to discussion about why the two amounts differed.</p>	

14. In the most effective lessons, teachers often presented new topics by challenging pupils to apply their mathematics to solve problems, drawing ideas from them and using probing questions to gauge their initial understanding and develop it. They sequenced learning carefully, helping pupils to make links to related areas of mathematics. They used visual aids and demonstrated ways of thinking that helped pupils to understand the methods they were learning and to overcome common misconceptions. The teachers listened to pupils carefully and observed their work throughout the lesson. They aimed to identify any potential misconceptions or barriers to understanding key concepts, and responded accordingly. They also emphasised the development and accurate use of technical language.

Prime practice: an interesting approach to a new topic	This lesson was a challenging introduction to three-dimensional applications of Pythagoras' theorem for a top set of Year 10 pupils. The approach enabled pupils to see how their existing knowledge of two-dimensional Pythagoras' theorem might be extended to the new three-dimensional context.
<p>The teacher provided models of a cuboid and a square-based pyramid made from straws. She asked the pupils to find the length of the diagonal of the cuboid and the height of the pyramid. After briefly inviting questions, she let the pupils get on with the task, circulating around the classroom to ensure they were all on a fruitful track. She intervened only if pupils appeared stuck when, by asking questions, she ascertained their thinking and moved it on. She did not steer pupils, at any stage, towards a particular method. This was a successful approach with alternative methods arising, which she discussed with the whole class later in the lesson.</p>	

15. In such circumstances, pupils become confident learners as they develop skills in articulating their thinking about mathematics. They are unafraid to 'have a go' at open-ended or unusual problems, and are willing partners in the processes of

teaching and learning. They learn to make sense of ideas, reason and justify their methods and solutions because discussion is a regular feature. Learning is therefore active and cumulative; they make good progress because they make connections with their existing knowledge and understanding. During the survey such pupils expressed clearly that they learnt well from others through discussion and informal pair work, and through explaining their thinking to the whole class.

<p>Prime practice: pupils persevering</p>	<p>This was a lesson on constructing triangles for low-attaining Year 10 pupils, who discovered for themselves why pairs of compasses are needed for constructing some triangles: it became a meaningful problem. (More usually, pupils are guided through the standard construction. As a result, they do not necessarily realise that to draw the triangle accurately without compasses is not possible.)</p>
<p>The pupils were asked to draw triangles of given dimensions for the three sides. They had access to rulers, pencils, protractors and pairs of compasses. They tried to carry out the task; the teacher gave them no extra advice or support at that stage. After 10 minutes, the pupils were concerned that they could draw two sides with the correct length but not the third. In essence, they had discovered the problem with the construction. One pupil used the compasses to draw some arcs but could not see how he could complete the triangle. The teacher used this pupil's ideas, demonstrating to the class what he had done, and asking them to think how it could help them. Again they worked in groups and, gradually, pupils were able to use the compasses effectively to draw the triangles. The fact that they had persevered with the task until they found the method, and realised the reason for it, gave them a very good understanding of how to draw triangles when the lengths of the three sides were given.</p>	

- Crucially, the best teaching seen concentrated on ensuring that every pupil was challenged throughout the lesson. This approach went beyond providing different work for groups and might be best described as pupil-centred or personalised. In such lessons, teachers made sure that no pupil was wasting learning time by being stuck, making incorrect first steps, or doing work that was too easy. They moved around the class, consciously watching and listening when giving instructions or quick questions to the whole class, while pupils worked independently or in groups. They intervened, posing questions to help pupils recognise misconceptions and errors for themselves and to understand the concept more fully. The questions the teachers used in lessons and set for homework were chosen carefully to reveal common misconceptions and to enhance conceptual understanding. This allowed the teachers to make efficient diagnostic assessment during the lesson and when they marked pupils' work.

Does teaching change according to pupils' different ages and abilities?

- Provision in the Foundation Stage involved children in a wide range of mathematical activities, typically a blend of free-choice play and focused learning. In the best examples, the environment was mathematically rich with opportunities to explore ideas and practise skills, and the outside areas were an extension of the classroom itself. Problem-solving, reasoning and numeracy were often integral to learning in other areas of the curriculum, for example songs and rhymes that involved counting,

creative patterns and designs made with beads or shapes, and family photographs used to discuss heights and ages. Many activities were practical, so learning was active and fun. An important element was the quality of adults' dialogue with the children; this was instrumental in their development. Young children often enjoyed trying to solve puzzles and problems, but only the better teaching required them to calculate or use the language of position rather than simply counting or naming shapes.

<p>Prime practice: language development</p>	<p>Learning mathematical language in a Reception class. The children's language and conceptual understanding were developed securely through a range of well planned activities that provided plenty of opportunity for them to use new words, make comparisons and reason.</p>
<p>The teacher was working outside with a group of five children. They were wearing hard hats and were 'working' on a construction site, designing and building a house for the Three Billy Goats Gruff. They had a superb range of equipment from which to choose, including planks of various lengths and wooden blocks of different shapes and sizes. The teacher participated in their play, asking well phrased questions to develop and assess their understanding of shape, weight and length, such as 'Can you find a shorter plank than that one?' and 'Is it heavier than the other one or lighter?' She recorded the children's responses on a prepared sheet.</p> <p>The activity was followed up well, using the interactive whiteboard and a program that showed pictures of different sized houses with three creatures of varying sizes alongside. The children were asked whether they thought the house would be better for the caterpillar, the dog or the giraffe, and were asked to explain why. All could offer good reasons: 'The giraffe's too tall, he wouldn't fit in.' 'It's a middle-sized house and the dog's middle-sized.' The children thoroughly enjoyed the activities, which also developed their gross motor skills, language, creative and social skills.</p> <p>Other child-initiated activities included role play in the toy shop where children were pricing items and buying and selling them. They had the idea of using coins and giving change even though they did not fully understand the mathematics: 'Here's 1p'; 'Thank you – you need 1p back.'</p>	

- Most primary mathematics lessons follow a three- or four-part structure: a starter activity as a whole class, with pupils often seated on the carpet in front of the teacher, followed by an introduction to the main learning. In the good lessons seen, teachers often used 'talk partners' effectively at this stage, with pupils collaborating to answer questions. Teachers then set pupils tasks, tailored to their group, which were usually attainment groups. Teachers used teaching assistants effectively, who generally worked with lower-attaining pupils or those who had learning difficulties, allowing the teacher to concentrate on another group. Well practised routines meant that the rest of the class worked independently within their groups. However, teachers varied in how successfully they picked up issues which arose around the class. This is one of the challenges that primary teachers face, namely to find quick ways of checking on the progress of pupils working independently while focusing intensively on a particular group. This was done most successfully when good

planning wove in key questions which allowed the teachers, as they moved around the class, to pinpoint quickly any misconceptions or errors. Lessons usually ended with the teacher gathering the class together to summarise what had been learnt.

19. In the secondary lessons, the most prevalent style was one where the teacher demonstrated a new mathematical method which pupils then practised. When this approach was used well, teachers developed pupils' understanding of why the method worked through explanations and activities. They selected a suitable range of questions so that pupils developed the necessary breadth of skills and understanding of the applicability of the method. A good example of this involved pupils choosing where to start exercises and the questions that challenged them; they enjoyed this responsibility, used it well and said how much better it was than wasting time on repetitive, easy questions. Notably, these good lessons avoided the common pitfalls of demonstrations that were limited to ways of remembering the method, followed by pupils working through similar examples. Such lessons do little to teach pupils how to use and apply mathematics; this style of teaching was noted in Ofsted's report on mathematics for 14–19-year-olds.⁵

Weaker factors: learning without understanding	In a Year 9 lesson, pupils learnt how to plot straight-line graphs but without appreciating the relationship between coordinate pairs and the equation of the graph, and with little idea how to interpret the gradient in terms of the slope.
<p>The teacher showed the pupils how to substitute three values for x in an equation such as $y = 2x - 3$ to obtain three pairs of coordinates. Pupils plotted the three points and joined them with a straight line. They rarely extended the line beyond these three points. A few pupils had difficulty because they did not realise that the numbers on the axes needed to be regularly spaced, and this led to dog-leg graphs rather than perfectly straight lines. The teacher had not checked quickly all pupils' work to ensure they had scaled their axes appropriately or to point out the problem resulting from not doing so.</p> <p>While pupils drew a selection of such graphs, the inspector asked some of them which other points were on the line. Most recognised only those where the line segment they had drawn passed through a point on the grid. They did not appreciate that the straight line consists of all points with coordinates (x,y) that fit the equation and no others, a principle that underpins much future graph-related work.</p> <p>Some pupils could identify the gradient in the formula because they had been told it was the coefficient of x, but not by looking at the graph. These pupils had no concept of what gradient meant in terms of slope. They could usually determine the intercept from memory as the constant term in the formula, but could not explain why it was necessary to put $x = 0$ into the equation to find the intercept on the y axis.</p>	
How might it be improved?	Inaccuracies in pupils' work could have been spotted quite easily, if the teacher had checked the work throughout the lesson, looking especially at axes to pick up on errors in spacing and at the line segments drawn to check for position, straightness and length.

⁵ *Evaluating mathematics provision for 14–19-year-olds* (HMI 2611), Ofsted, 2006; www.ofsted.gov.uk/publications/2611

	<p>For better learning, the teacher might:</p> <ul style="list-style-type: none"> ■ pose questions to check understanding, for example whether points such as (5.5, 8), (-10, -17), (3, 2), (100, 197) lie on the line with equation $y = 2x - 3$ ■ focus more on the meaning of 'gradient' and how it might be read directly from the graph as the increment in y for unit increment in x. <p>Learning could be extended by asking pupils what lines might be parallel to $y = 2x - 3$, or how could they use it to draw the lines with equations $y = 4x - 3$ and $y = -2x - 3$?</p>
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20. Setting by ability is common in secondary schools and, increasingly, in the upper years of Key Stage 2. During the survey, marked differences were found in the quality of teaching according to how secondary schools grouped pupils. Sets of high-attaining pupils and mixed-ability classes fared best. Sets of middle-attaining pupils were the least likely to receive good teaching, even though these groups include the 'borderline' pupils whose success or otherwise determines progress towards the Government's targets and schools' positions in published tables of performance data. During the survey, the highest proportion of inadequate lessons (15%) was in sets containing low-attaining pupils in secondary schools. However, factors such as the expertise of non-specialist and temporary teachers, as well as pupils' disaffection with learning mathematics, also contributed.
21. Schools rarely evaluate the impact of their structures for groupings on pupils' attainment and self-esteem. Typically, it is the pupils' sets that define the mathematics they will subsequently learn. This can place an artificial ceiling on their attainment. Occasionally, schools try out different models of grouping, such as single-sex GCSE sets, some of which meet schools' particular circumstances, or pupils', or both, better than others. Sometimes primary schools invest in additional staff to create extra teaching groups in Year 6, allowing teachers to focus strongly on particular topics in readiness for the national tests.

Using assessment in teaching and learning

22. Teachers' use of assessment continues to develop, often as part of whole-school programmes. It is generally more effective in primary schools, where teachers know their pupils better and can draw on assessment skills from other subjects and apply them to mathematics. Despite recent initiatives, assessment for learning remains one of the most important areas of inconsistency and weakness in many secondary departments. Part B discusses this in more depth.
23. The frequency and quality of homework varied widely. The tasks set for homework rarely captured pupils' imagination or extended their learning, concentrating instead on pupils practising taught skills. While this is important, since pupils need to be fluent in skills if they are to have the intellectual space for thinking when they tackle more complex or unusual problems, it should not be pupils' only experience of independent work. An example of homework being used constructively was the

setting of a small amount after every lesson. This helped pupils to reflect and build on what they had learned and, in the following lesson, to ask for help with any difficulties, ensuring that they did not fall behind. Some teachers used the problems posed in the closing minutes of a lesson creatively, requiring pupils to work on them before the next lesson, when they became the starting point of that day's learning. One pupil said, 'We don't get any homework in our set. We were supposed to use [an online revision service] but we didn't, so the teacher gave up.'

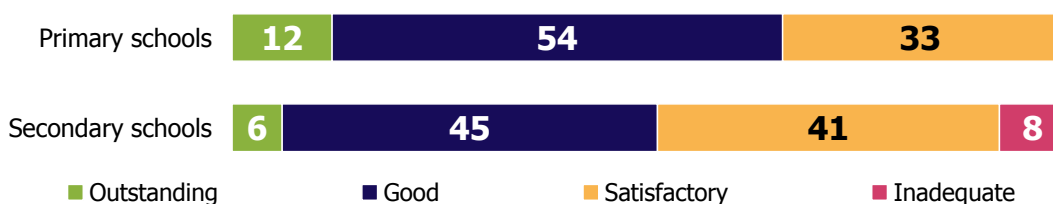
Prime practice: primaries involving parents	Two examples of primary schools seeking to involve parents in supporting their child's learning in mathematics: a games library and a website.
<p>A school was concerned that pupils' learning and confidence in applying numerical skills were not being reinforced sufficiently, especially at home. Formal homework was not seen to be the answer because the children were young. The teachers had the idea of a 'games library', accessible to all parents each week. Games were exchanged in the same way as library books. The games were colour-coded for age and ability so parents knew which were suitable for their children. The introduction of the library was considered to be a success story. Parents acted as librarians and the library was open on fixed days at the end of the afternoon, which was convenient for parents. The headteacher said that children loved having such a wide variety of games and that there had been a noticeable improvement in the progress and confidence of those who played with them with their parents. In addition, the children were becoming familiar with the idea of libraries as a valuable resource for themselves and adults.</p> <p>Another school took guidance for parents and carers one step further than the usual information evenings. The pupils devised guidance on calculation strategies, such as doubling and halving. This was uploaded onto the school's website where it was supplemented by information on methods and progression in calculation.</p>	

24. Despite the development of school policies and guidance for staff, the quality of marking remains variable. It was rarely consistently good across a whole primary school or secondary department in the survey. The best marking was diagnostic and selective, focusing on areas where it would make the most difference. It provided pupils with helpful feedback, enabling them to understand the source of their errors and identifying what they needed to do to improve, although pupils did not always follow up the advice. Some teachers tried to mark all the work pupils had done, but a more thoughtful approach that identified what should be marked by the teacher and what could be marked by the pupils might have made the marking more constructive for the pupils and manageable for the teachers.
25. Subject leaders and senior leaders did not tackle inconsistencies in the quality of marking sufficiently robustly, for example by following up instances when work was simply not marked at all. Typically, the marking seen during the survey indicated whether pupils' answers were right or wrong but did not spot misconceptions or help them to improve. A key question for checking the effectiveness of marking might be: 'If a pupil were to tackle the same piece of work again, could he or she do it any better as a result of the marking?' If not, has the marking served any purpose?

What is not good enough about 'satisfactory' teaching?

26. A substantial amount of teaching is no stronger than satisfactory and, in these lessons, pupils do not learn as quickly as they might. Annex B compares characteristics of satisfactory teaching in mathematics with those of good teaching. Teaching was satisfactory in 33% of primary mathematics lessons and 41% of secondary ones.

Figure 4: Quality of teaching in mathematics lessons in the schools surveyed (percentages of lessons)



Figures should be treated with caution due to sample sizes.
Percentages are rounded and do not always add exactly to 100.

27. Typically, the teachers had established clear routines and pupils paid attention to their explanations. A common shortcoming, however, was that teachers failed to give sufficient attention to whether all the pupils had understood the work. They usually checked whether pupils were completing the exercises; answers were often read out and marked right or wrong by the pupils, particularly in secondary lessons. In isolation, this checked the acquisition of a skill but did not verify whether pupils had understood. Errors or misconceptions were not always exposed: some pupils got the answers from their peers, others altered their answers to the stated correct ones, and some had not progressed far through the exercise. Some teachers referred frequently to the level of the work and how it related to examination requirements, which could be helpful for the pupils, but they did not monitor the quality of the learning or assess whether the work made sense to the pupils. The teachers did not show enough urgency in checking whether each pupil had started the work correctly, had shown any of the expected misconceptions or was being challenged enough. Planning for these lessons tended to focus on developing pupils' knowledge and skills but not explicitly on promoting their understanding.

Weaker factors: rote methods	A Year 10 lesson on finding the fraction of an amount. The teacher had a clear view of the types of question pupils needed to be able to do to be successful in the foundation tier of the GCSE examinations.
<p>The teacher showed the pupils how to calculate $\frac{3}{4}$ of £10.80 by dividing by 4 and multiplying by 3. He did not explain why. One pupil called out, offering her own method. The teacher discouraged her, but she insisted on telling him, even though he didn't want to know that 'You halve it, then halve it again, and add the two halves together.' She probably meant that she would add the first $\frac{1}{2}$ to the halved half, $\frac{1}{4}$, to make $\frac{3}{4}$ but the teacher did not pick up on this clue. Instead he repeated his method of dividing by the denominator and multiplying by the numerator, all as one calculation.</p> <p>When pupils tackled similar questions, many of them reached the right answers, but none of the pupils to whom the inspector spoke could explain why they were dividing by the</p>	

denominator and multiplying by the numerator. Some pupils could understand why dividing by the denominator gave one part and, coaxed for the answer, why they would then multiply by the numerator.

The teacher moved around the classroom while pupils worked steadily through the exercise, helping those who were stuck by demonstrating the same method again.

Discussing the lesson afterwards with the inspector, the teacher could see how he had emphasised the technique without any reference to understanding. He commented to the inspector the next day that the discussion had made him reflect critically on the methods he often used in his teaching.

How might it be improved?

A different starting point might have been to use an easy example that pupils could do in their heads, say $\frac{3}{4}$ of £20 or £10, and then probe how they worked it out. Listening to their responses could provide insights into their thinking, and the teacher could use their explanations as the starting point for developing a method.

Learning would have been better if pupils had been enabled to make the connection between finding the fraction, $\frac{1}{4}$, of something and dividing it by 4 to give four equal parts. Practical equipment might help, although most pupils find money easy to understand.

28. Satisfactory lessons were also characterised by the teacher doing most of the talking, emphasising rules and procedures rather than concepts or links with other parts of mathematics, sometimes even omitting an example in their textbook that illustrated why the method works. Often, when teachers explained in great detail what to do, pupils were left with relatively little time to work independently on the exercises. In total, this 'teacher talk' constituted a substantial proportion of pupils' time for learning mathematics. In the weakest cases, it was not unusual for a teacher to follow a starter activity and 20 minutes of explanation with an exhortation such as that heard in one unsatisfactory lesson: 'Now, I want you to do some questions yourselves. Hurry up! You have seven minutes.'
29. Pupils can find such teaching methods frustrating. Higher-attaining pupils told inspectors that they found much of the time given to whole-class explanation undemanding, especially when other pupils asked for additional clarification. Often, particularly in secondary and upper Key Stage 2 classes, pupils listened attentively but passively. Their role was confined to watching the teacher provide examples, copying examples from the board, or both, followed by answering routine questions from a worksheet or textbook. If there were any interesting questions with challenging twists, it was generally only the fastest workers who reached them and had their understanding challenged. This made it difficult for pupils to make sense of the bigger picture or solve problems other than routine ones. In secondary lessons, pupils often sought help the moment they became stuck, showing dependency on their teachers, whereas, in lessons in the primary schools visited, pupils were reluctant to seek help if their group was not the focus of an adult's attention. This sometimes led to good discussion amongst them, but often one pupil copied from another or the pace of work slowed.
30. For many pupils, mathematics consists of a regular diet of broadly satisfactory lessons. Despite some strengths, such as the management of pupils' behaviour and

clear instructions, a common weakness was that aspects of learning were transient or superficial. In many cases, pupils simply completed exercises in textbooks or worksheets, replicating the steps necessary to answer questions in National Curriculum tests or external examinations. Success too often depended on pupils remembering what to do rather than having a secure understanding that underpinned their thinking and application of techniques.

31. Furthermore, by practising only one method at a time, pupils were not gaining the confidence and intellectual flexibility they need. This also had the effect of fragmenting the subject, because it was presented as a collection of apparently arbitrary rules that had to be memorised. The rules were sometimes incomplete or confusing. Occasionally, pupils found it difficult to distinguish between rules that have a basis in mathematical necessity, such as, 'Turn the fraction over and then multiply', and the teacher's own rules such as, 'Always underline the date'.

Weaker factors: unhelpful rules	Teachers usually introduce rules to help pupils remember particular results or steps in methods. However, few are always true and many are never convincingly developed with pupils so that they understand the particular context within which a rule might be used. Here are three examples.
<p>(a) 'To multiply by 10 you add a nought' but $3.4 \times 10 \neq 3.40$. Discussion about place value is the most powerful way of tackling multiplying by 10.</p> <p>(b) 'Always measure from the end of the ruler' but this doesn't always work, and is a common mistake young pupils make when learning to measure. Another error is that they measure from 1 on the scale. The emphasis should be placed on measuring from 0, which is often at the end of a tape measure but the scale on most rulers starts a little way in from the end of the ruler.</p> <p>(c) 'Two minuses make a plus' $-5 \times -3 = +15$ but $-5 + -3 \neq +8$. This rule is an inaccurate simplification of a generalisation. Incorrectly applied 'rules' on signs and operations are the source of many errors for secondary pupils in work on number and algebra, usually because the 'rule' is learned without understanding and they do not take into account the different contexts of the operations of multiplication and addition, and the positive and negative states.</p>	
How can it be improved?	Where it is considered that rules might be useful, they should be unambiguous and developed with the pupils. The unthinking use of rules should be discouraged.

32. A lack of depth in pupils' knowledge, skills and understanding can result from teachers moving on to the next topic too quickly. Other factors include gaps in earlier learning, as well as pupils' reliance on formal written methods and a reluctance to use informal or mental strategies which are sometimes more efficient. The best teaching gave time for pupils to think; too often, however, teachers or teaching assistants stepped in at an early stage and did the thinking for the pupil. Sometimes, pupils' passivity turned into low-level disruption and a resistance to thinking. This was most commonly a feature of middle or low-attaining sets in secondary schools.

Teachers' common strategy in these instances was to set undemanding work to keep pupils busy.

33. Most of the inadequate lessons observed were in secondary schools. Their format was similar to that of the typical satisfactory lesson described above but less effective, principally because the work was not matched well enough to pupils' needs or interests, and this limited their progress. The teacher's explanations were often stilted or unclear, interrupted by pupils calling out or by the teacher breaking off to remonstrate with restless pupils. Controlling behaviour replaced learning as the focus of the lesson. Managing pupils' behaviour can become a persistent issue in the longer term. Sometimes, teachers were unsure of the mathematics involved and so explanations to pupils were confusing or incorrect.
34. The main difference between good and satisfactory lessons is in teachers' expertise in mathematics and how they use it to promote the learning of all pupils. Weaknesses in mathematical knowledge and pedagogy often have a limiting effect, particularly on assessing and developing pupils' understanding. This represents the biggest challenge in raising the quality of teaching, and thereby standards. Part B explores this more fully.

Help from additional adults in lessons

35. Teaching assistants in primary schools generally work in partnership, often seamlessly, with class teachers. There is less consistency and continuity in working partnerships between teaching assistants and teachers in secondary schools. In both phases, teachers could make greater use of teaching assistants when working with the whole class, for instance during starter activities. Examples from good lessons included teaching assistants who recorded responses made by particular pupils and who used prepared prompts or questions to support those with learning difficulties. There is also scope to improve teaching assistants' mathematical interactions with pupils during group and individual work, so that they become more skilled in asking questions rather than telling pupils what to do. At times, the use of teaching assistants and volunteer helpers such as sixth formers or parents, while often motivating, can compound weak teaching methods, especially when the teaching assistant or helper does too much of the thinking and doing for the pupil. Many schools had invested in training for their teaching assistants; this was proving beneficial in supporting pupils' learning in mathematics. In a few schools, the training was leading to qualifications, such as a degree in early years education.
36. Assigning teaching assistants to secondary mathematics departments is increasing. This accords with the Government's drive to provide a higher-level teaching assistant for all mathematics departments in the future. This is allowing them to improve their expertise in mathematics and is working effectively in some schools.

The curriculum and other activities

What do pupils study in mathematics?

37. The Foundation Stage curriculum guides the mathematical development of children aged three to five years.⁶ They learn about numbers, using them to count and calculate, and about shapes and simple forms of measurement. For pupils aged five to 16, the mathematics curriculum is defined by the National Curriculum programmes of study. Pupils are taught about number and shape, space and measures and, as they progress, they also learn about handling data and algebra. The programmes of study also specify the mathematical processes that comprise 'using and applying mathematics'. Crucially, these processes are given far greater weight in the most recent National Curriculum, which is to be implemented in stages over the next few years.
38. During Key Stage 4, most pupils take GCSE mathematics. An increasing number, mainly the more able pupils, also study GCSE statistics and a small minority take a GCSE in additional mathematics. Some schools enter pupils for adult numeracy courses. A minority encourage high-attaining pupils to take a GCSE early and these pupils often, but not always, subsequently progress to AS mathematics in Year 11. Having achieved a grade B or C, some pupils give up the subject. One Year 11 pupil who was studying an AS unit reflected, 'I think I would have been better off trying to get an A* in Year 11 than what I am doing now'. In the most effective schools, the mathematical pathways pupils follow are thoughtfully matched to their individual needs, abilities and aspirations. However, this is not the case often enough and concerns expressed previously remain.⁷
39. In the sixth form, a wide choice of mathematical subjects exists at A level, all of which have a common core of pure mathematics. Although more students are now studying mathematics and further mathematics at AS and A level, the vast majority have attained the highest grades in GCSE mathematics. The subject struggles to attract and retain students from a more moderate academic background; mathematics is still viewed as more difficult than most other subjects and analysis of results shows this to be the case. However, a few schools and colleges manage to break this mould, usually through high-quality provision that is tailored to meet students' diverse needs.
40. There have recently been green shoots of improvement, most marked in primary schools, in the quality of the mathematics curriculum and other activities. It was good or better in 60% of primary schools inspected but only 42% of the secondary schools. Positive developments in the mathematics curriculum in some primary

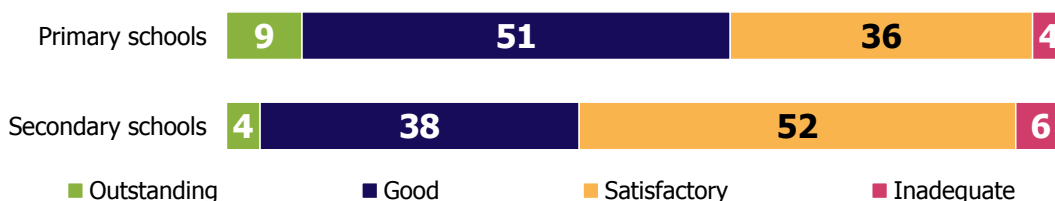
⁶ *Curriculum guidance for the foundation stage* (QCA/00/587), QCA, 2000. From September 2008, the *Statutory Framework for the Early Years Foundation Stage* sets the standards for learning, development and care for children from birth to five years (DfES 00013-2007BKT-EN), DfES, 2007.

www.standards.dfes.gov.uk/primary/publications/foundation_stage/63593/

⁷ *Evaluating mathematics provision for 14-19-year-olds* (HMI 2611), Ofsted, 2006; www.ofsted.gov.uk/publications/2611

schools included problem solving as an integral part of lessons and the use of ICT to support learning.

Figure 5: Quality of the curriculum in of mathematics in the schools surveyed (percentages of schools)



Figures should be treated with caution due to sample sizes. Percentages are rounded and do not always add exactly to 100.

41. Where it was possible to make comparisons, the quality of the mathematics curriculum itself was often worse than that for the secondary school as a whole. This was usually because of weaknesses and inequalities in the provision for 'using and applying mathematics'. In the schools in which staff did not share an effective underpinning philosophy about mathematics, it was frequently the case that the richness or otherwise of pupils' experience in the subject depended on individual teachers rather than on the school. Sometimes, this unevenness of experience resulted in pupils not receiving their full entitlement to the mathematics curriculum. Schools did not readily recognise that such inequalities meant they were not as inclusive about their provision for mathematics as they generally considered themselves to be.

The quality of planning in mathematics

42. Most primary schools in the survey had used the Primary National Strategy framework effectively to plan their mathematics curriculum, often tailoring it to the school's particular context. This helped to structure pupils' learning about number, shape and space, measures and handling data, as outlined in the National Curriculum, and helped to plan for progression in each strand. The revised framework, implemented from autumn 2007, importantly places greater emphasis on developing pupils' skills in 'using and applying mathematics'. However, it is too early to evaluate the difference this is making. Many primary schools, especially in the Foundation Stage and Key Stage 1, made good use of resources and practical activities to make learning enjoyable and help pupils to grasp important concepts. The oldest pupils, however, tended to have far fewer 'hands-on' experiences. One of the schools visited was developing an interesting approach to teaching mathematics through story themes, such as *Santa's Little Helper*, working as part of a cluster of schools with the local authority's mathematics adviser.
43. Secondary schools commonly used a range of textbooks, examination specifications and National Strategy materials to guide planning in mathematics. These generally reflected the content defined by the National Curriculum programmes of study and so provided adequate coverage of number, algebra, shape and data handling. This

ensured that the vast majority of pupils studied mathematical content that was broadly appropriate for their age.

44. In both phases, planning for and tracking pupils' progress in the key processes of 'using and applying mathematics' remain weak and, consequently, are the most underdeveloped elements in pupils' learning. This is discussed further in Part B.
45. The best schemes of work included guidance on approaches, interesting activities and resources that help nurture pupils' understanding. They were seen as living documents, subject to regular discussion and review, which helped staff to develop their expertise. Primary schools often provided guidance, for example on approaches to and progression in calculation, within their mathematics policies rather than in schemes of work. Some supplemented this with guidance from other sources, including the Primary National Strategy and their local authority. However, these separate policies and guidance were not always kept up to date or implemented consistently.
46. Good schemes of work were rare in secondary schools. It was not uncommon for teachers to use only examination specifications and textbooks to guide their lesson planning, focusing on content rather than pedagogy. Few schemes included guidance on matters such as the most effective teaching approaches, how to meet the full range of pupils' needs or on what constitutes an appropriate level of challenge. They provided insufficient support for teachers who were at an early stage in their professional development or for staff who were not mathematics specialists.
47. In many secondary schools, apart from adaptations needed because of changes in examination specifications, there has been little progress in developing the mathematics curriculum since the Key Stage 3 Strategy's sample medium-term plans several years ago.⁸
48. A small number of schools have used the introduction of the new two-tier GCSE to re-energise their Key Stage 4 schemes of work. This was not the case more generally. Instead, departments often simply identified topics added to or removed from previous specifications. Because of a lack of guidance, many teachers believed that the whole specification needed to be taught, irrespective of pupils' attainment at Key Stage 3. Of particular concern is that some departments intend to 'play safe' by entering relatively able pupils for the foundation tier GCSE examination, thereby placing a ceiling on their achievement.

Schools' use of intervention and other strategies

49. Schools use a range of strategies to help pupils catch up if they are at lower points than their peers, are in danger of falling behind, or need an extra boost to reach the standards expected of their age. Intervention happens at different levels – whole class, small groups and for individual pupils – and in different ways, including in-class

⁸ *Key Stage 3 National Strategy sample medium-term plans for mathematics* (Ref 0504/2001), DfES, 2001. www.standards.dfes.gov.uk/secondary/keystage3/all/respub/ma_samplepln

support, withdrawal from mathematics lessons, withdrawal from other lessons, and at various times of the school day.

50. In the best cases, intervention strategies were used thoughtfully and in a short-term focused way, as in a primary school where a teacher pinpointed the specific difficulties a small group of lower-attaining pupils was having. The deputy headteacher taught these pupils for a short period and, as soon as they were confident on the topics, they rejoined their mixed-ability class.
51. More generally, intervention strategies were not consistently well understood or thought through to ensure gaps were identified and bridged effectively so that gains in progress were real and sustainable. There were exceptions, but many intervention strategies concentrated on the skills pupils needed to answer the questions in tests. Some pupils' experience of learning mathematics was largely defined by consecutive implementation of intervention strategies: they were permanently trying to catch up.
52. In 2006/07, the Secondary National Strategy piloted an intervention programme called 'Study Plus' which focused on pupils who were in danger of falling short of grade C at GCSE.⁹ It took up time that would normally have been given to a GCSE option. Following positive feedback, it was rolled out in 2007/08. The programme is unusual because it promotes a markedly different approach to teaching and learning. The materials are based on substantial real-life problems that are explored in detail. Schools are encouraged to adapt the sample units to meet the individual needs of the relevant pupils. Some pupils who were participating in the pilot said the problems were interesting and helped them to make sense of the mathematics involved. They could see connections with other subjects, such as science, and with their lives outside school. One pupil explained, for example, how the work she had done recently on cars had been useful when she and her parents were discussing selling the family car.
53. In addition to intervention strategies, most schools run revision sessions for several weeks – and sometimes months – before the national tests and examinations. Rather than finding and tackling weaknesses in core provision robustly, a growing dependence on strategies to boost pupils' performance in the short term, particularly ahead of tests, is emerging. A danger exists that such practice will become the norm; it is presently too readily accepted as such by teachers and pupils alike. Schools work hard to get as many pupils as possible to reach key performance thresholds in Years 6, 9 and 11 (usually Levels 4, 5 and grade C respectively). Schools focus on the pupils who are in danger of not achieving these, but this common focus on borderline grades or levels can result in the lowest- and highest-attaining pupils receiving relatively little attention.

⁹ *Study plus* (Ref DFES-03987), DfES, 2007. www.standards.dfes.gov.uk/intervention/home.html

The contribution of information and communication technology to the mathematics curriculum

54. Several years ago, inspection evidence showed that most pupils had some opportunities to use ICT as a tool to solve or explore mathematical problems.¹⁰ This is no longer the case; mathematics makes a relatively limited contribution to developing pupils' ICT skills. Moreover, despite technological advances, the potential of ICT to enhance the learning of mathematics is too rarely realised.
55. In the survey schools, primary pupils had some opportunities to use ICT in mathematics but there was scope to develop this further. Typically, one or two pupils at a time used a couple of computers in a classroom to practise basic skills. Many schools had a separate computer suite, where lessons across a range of subjects enabled pupils to learn how to use ICT as a mathematical tool, for example using spreadsheets to generate number patterns or present statistical information, but using it to enhance learning in mathematics was more limited.
56. In the secondary schools, the two main problems were the lack of ICT resources and weaknesses in identifying suitable activities at key points in schemes of work. Consequently, work in mathematics was not supporting pupils' preparation for their future lives in the way that it should. The lack of access to ICT facilities was due, in the main, to the growth of ICT as a discrete subject. A small number of the departments visited overcame this by having portable laptop computers.
57. The interactive whiteboard featured in many (but not all) primary and secondary classrooms, bringing positives and negatives to teaching and learning. Good practice included the use of high-quality diagrams and relevant software to support learning through, for example, construction of graphs or visualisation of transformations. Pupils enjoyed quick-fire games on them. However, many of the curricular and guidance documents seen did not draw sufficient attention to the potential of interactive whiteboards. Additionally, too often teachers used them simply for PowerPoint presentations with no interaction by the pupils.

Prime practice: ICT	Use of the interactive whiteboard and internet to scale a picture from very tiny to extremely large
A Year 7 class, working on scales, was shown a website using the interactive whiteboard where a picture was scaled from 10^{-16} metres to 10^{16} metres, that is, 10,000,000,000,000,000 metres. The pupils were amazed; they became animated and excited, discussing the effect of scaling by powers of 10. The teacher posed questions, asking pupils, in pairs, to describe and explain their thinking. Some presented this from the front of the class with their peers critically appraising it in a lively discussion.	

¹⁰ *ICT in schools* (HMI 264), Ofsted, 2001; www.ofsted.gov.uk/publications/264 and *ICT in schools* (HMI 423), Ofsted, 2002; www.ofsted.gov.uk/publications/423

Although these reports discuss the variability in the use of ICT in mathematics, features which were typical of satisfactory and better practice at the time are less evident now.

58. A negative effect of interactive whiteboards was a reduction in pupils' use of practical equipment: software is no replacement for hands-on experience, for example in measuring angles and lengths. Teachers generally underused practical resources and games to develop pupils' understanding of mathematical ideas and help them to make connections between different topics.

Weaker factors: visualisation	A Year 1 lesson about the properties of three-dimensional shapes was based on images displayed on the interactive whiteboard but gave no practical hands-on experience of the solids.
A teacher used an interactive whiteboard to teach Year 1 pupils about three-dimensional shapes. The pictures of the shapes caused confusion, between spheres and circles for example. Although pupils enjoyed a matching activity using the interactive whiteboard, they did not develop knowledge and understanding of the properties of three-dimensional shapes, such as the nature of the surfaces of a cone. The teacher did not adapt the teaching to take account of pupils' responses that showed their difficulties in using the two-dimensional representation of the three-dimensional shape.	
How might it be improved?	Pupils would benefit from handling a range of real shapes so that they could feel and see the difference between two-dimensional and three-dimensional shapes. They could be encouraged to use their knowledge of properties of two-dimensional shapes to help describe the three-dimensional ones.

59. The change in rules that restricted the use of graphical calculators in AS and A-level examinations from 2000 had a severely negative impact on their use as a tool for teaching and learning. There has been limited recovery from this, with many teachers reverting to former methods for teaching topics such as graphs and transformations, for instance, thereby missing opportunities to exploit the power of hand-held technology in promoting students' understanding.
60. Secondary school pupils are making increased use of individual online help both in school and at home, finding it particularly useful in the run-up to national assessments. However, the benefit is often short lived. Few programs have conceptual explanations or demonstrations and most examples practised immediately before the self-assessment are similar to those in the assessment itself.

Links between mathematics and the wider curriculum

61. In both phases, teachers missed opportunities to develop numeracy across the curriculum in meaningful ways and these were rarely explicit in planning. In a few lessons seen, teachers used good examples of mathematical applications drawn from other areas of the curriculum. More often, in discussion with inspectors, pupils struggled to talk about how they used mathematics in other subjects and about its wider application to work and life beyond school. Work on improving the contribution of mathematics to other subjects, and vice versa, showed some promise in secondary schools a few years ago. However, this has stalled in the face of numerous curricular changes in the subject, such as the multiple revisions to GCSE and advanced-level specifications.

62. In a few schools, cross-curricular and other focus or themed days added enjoyment and value to many pupils' learning in mathematics, particularly in primary schools, for example a 'maths and art week' in one school and a team problem-solving day in another. However, during the last year, inspectors have seen little extra-curricular provision in mathematics. A few schools offered optional ICT clubs that had a mathematical dimension, but few mathematics clubs were evident, other than those aimed at improving performance in tests and examinations.

Prime practice: real enrichment	Year 6 pupils investigate projects and bid for money from governors in the style of a popular television programme.
<p>Groups of Year 6 pupils thought up ideas, consulted the rest of the school, and then planned their projects, including a healthy eating tuck shop and outdoor play. They carried out research through questionnaires, collating their findings, using ICT very well. They researched costings, knowing they were expected to prove best value by comparing prices. The pupils devised criteria to ascertain which projects went forward to the judging panel, which comprised five governors, the chair of the Friends of the School and the headteacher. For this, they created presentations that gave a rationale, statistical analysis and justification for their project, including graphs and charts for visual impact, to convince the panel to part with their money.</p> <p>All the groups were granted at least some of their funding and soon several schemes were in train. The pupils overcame practical problems as they arose, for example acquiring old supermarket trolleys to customise into a tuck shop, helped in this design and technology project by a local secondary school. The project met its aims including the application of skills in calculation, problem-solving, communication, collaboration and ICT in a real-life context. Pupils enjoyed the contribution they made to the projects.</p>	

63. For pupils identified as gifted and talented, the mathematics curriculum was often enhanced by a range of stimulating extra-curricular activities. These included participating in national and local mathematics competitions and master classes, sometimes arranged through links between partner secondary and primary schools. However, such a demanding level of challenge was not often replicated in lessons. This was apparent even in otherwise effective schools and was a key area for improvement in a fifth of the primary schools inspected in the survey. In both phases, the best planning for high-attaining pupils did not require them to complete the same or similar work to their peers before progressing to work that was more demanding but, instead, presented them with challenging work at the start, and included problems set in new contexts and open-ended tasks which deepened and extended their knowledge and understanding.

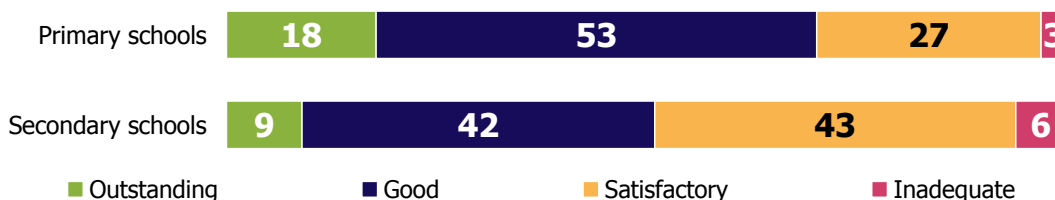
Leadership and management

The quality of leadership and management

64. In general, when judging the leadership and management of mathematics, inspectors consider the combined impact of senior staff and subject leader. The

quality was good or better in 71% of the primary schools and 51% of the secondary schools visited.

Figure 6: Effectiveness of leadership and management in mathematics in the schools surveyed (percentages of schools)



Figures should be treated with caution due to sample sizes. Percentages are rounded and do not always add exactly to 100.

65. The quality of subject leadership and management in secondary schools has improved in the last two years. This is an achievement in the face of the difficulties many schools experience in appointing suitably qualified and experienced staff to lead the mathematics department. In these circumstances, some headteachers have resorted to solutions that include job-shares, persuading staff to step up to the role, placing mathematics within a faculty under the leadership of a head of another subject, and offering the role of head of mathematics alongside a more senior whole-school role to attract applicants. Schools will need to evaluate such strategies carefully. An innovative approach, funded jointly by a local authority and six secondary schools, involved the appointment of an adviser responsible specifically for supporting the heads of mathematics departments in developing teaching and learning, and exploring recruitment and retention with senior staff.
66. Many schools evaluated accurately the effectiveness of their work in mathematics and identified strengths and areas for development. In primary schools, the quality of mathematics provision generally matched the rest of the school's work. This was not the case in the secondary schools visited, where work in mathematics was often weaker than the rest of the school's work overall and rarely a notable strength. Many senior staff recognised this. It highlights the need to improve provision for mathematics at a time when including mathematics and English in one of the high-profile measures of performance at GCSE level has already increased the pressure on mathematics departments. Overall, however, senior staff in secondary schools did not give sufficient consideration to supporting and developing either new or established leaders of mathematics.
67. The collaborative support of senior staff, often the headteacher, for the subject leader helped to strengthen the effectiveness of leadership and management in many primary schools. This was particularly important for new or inexperienced subject leaders, as was the case in a quarter of the schools visited. Other than for professional satisfaction, there is a lack of incentive for primary teachers to develop their expertise in mathematics in order to be better placed to support their colleagues and raise standards. Since the national reforms to responsibilities for teaching and learning, the leadership of subjects is no longer as hierarchical as

previously, when leadership of mathematics and English tended to be regarded as more senior posts and often carried a responsibility point.

68. Schools use a number of strategies for internal and external monitoring of mathematics. These include programmes of monitoring by senior staff, and visits from local authority staff, independent inspectors and consultants, or a mixture of these. The monitoring of teaching however does not consistently place enough emphasis on the impact of the teaching on pupils' progress, both lesson by lesson and over time. In many cases, the distinction and the relative emphases within pupils' learning between acquiring skills and developing understanding were not made by those observing the lessons. Yet this is crucial if the areas for development are to improve teaching and, through this, pupils' understanding of mathematics. Professional development was consequently often not focused sharply enough or followed through adequately.

The characteristics of good and weaker subject leadership

69. The effective leaders used data strategically. Robust monitoring, a characteristic of good management, led to the accurate identification of strengths and areas for development but the best leaders took this one step further. They used the outcomes of monitoring and analysis of test results to inform approaches to teaching and learning and the development of the curriculum. They also used professional development opportunities to disseminate and build on good practice and to tackle areas of inconsistency and weaknesses. Effective practitioners helped colleagues to develop aspects of their work. Occasionally, this included developing teachers' knowledge of mathematics, as well as how it might be taught. Teachers' readiness and commitment to giving and receiving such support was a hallmark of the school or department's ethos. Such an approach was seen not simply in high-achieving schools but also often in those working hard and effectively to improve, sometimes in challenging circumstances.
70. Conversely, weaker leaders tended to rely heavily on their assumptions about the strengths of individual teachers, the degree of consistency, and the extent of teamwork among staff. For example, subject leaders sometimes made assumptions about teachers' use of activities to support 'using and applying mathematics'; some senior managers interpreted quiet individual work on textbook exercises as good learning in mathematics. While informal strategies provided some useful insights, they did not reliably uncover weaknesses and pinpoint areas for development. Monitoring which was insufficiently systematic and robust generated too rosy a view of provision and little impetus for improvement.
71. Good leadership was reflected in consistent approaches across a school or department, such as in developing mathematical language and attention to its accurate use. The best examples of this were in primary schools, where staff emphasised the development of pupils' oral responses as a way of overcoming weaknesses in their communication skills.
72. The quality of departmental improvement planning varied widely; the best tackled identified shortcomings and areas for development, linking to whole-school priorities

where relevant, with clearly defined actions, and measurable success criteria. A positive development in departments which were effective and improving was the use of meeting and planning time to discuss teaching and learning and share ideas.

73. Many schools had suitable structures for assessing the standard of pupils' work and for tracking their progress against long-term targets. While senior managers and subject leaders used these systems increasingly well to identify pupils who were in danger of underachieving and for whom intervention was appropriate, in many cases they did not take the opportunity to raise questions about the effectiveness of teaching, learning and the curriculum.
74. Primary schools were generally ahead of secondary schools in their use of tracking data to set termly or half-termly curricular targets, typically for groups of pupils of similar attainment, and in the way teachers used assessment information in planning activities to meet pupils' varying needs. Pupils' involvement in their curricular targets, though, was less well developed.
75. In secondary schools, the appropriateness of targets against which pupils' progress was measured and the frequency and accuracy of assessments were often problematic. Although the principle that pupils' individual targets should be challenging yet achievable was widely accepted, schools did not take enough care to ensure that they were; for instance, a GCSE target of grade C for a pupil who had already achieved Level 7 at Key Stage 3 was inappropriate because it represented no progress during Key Stage 4; a target of grade A would have been suitably challenging. Occasionally, target grades carried undue weight in determining pupils' sets, the mathematics topics studied, and the GCSE tier for which pupils were entered. Teachers' expectations can be influenced positively and negatively by pupils' targets, so it is important that the targets are appropriate.
76. Recently introduced materials known as 'Assessing Pupils' Progress' aim to help teachers build a profile of pupils' attainment, track their progress and use assessment information diagnostically.¹¹ Early signs are that these are being positively received, especially where teachers work collaboratively.

Training for teachers and subject leaders

77. Middle managers receive training which is predominantly generic. It focuses on developing robust monitoring activity that mirrors that of senior management. In particular, there is a strong focus on tracking pupils' progress and subsequent intervention with targeted groups, and on checking compliance with policies, such as those on marking and lesson planning. These management functions are important but there is an urgent need to give far greater weight to ensuring that subject leaders focus on improving the quality of teaching and learning and on the

¹¹ *Assessing pupils' progress in mathematics at Key Stage 3* (Ref. 00007-2007DOM-EN), DCSF, 2007. The 2008 Key Stage 2 materials and guidance on assessing pupils progress can be found at www.standards.dcsf.gov.uk/primaryframework/assessment/app/

curriculum that pupils receive. To do this effectively, subject leaders need good subject expertise, which is not the case in many schools. This raises questions about how this might be achieved and whether each school and local authority is in a position to identify exactly what needs to be done and then to help drive the improvement. Very recent developments from the Training and Development Agency for Schools and the National Centre for Excellence in the Teaching of Mathematics around courses for subject leaders and support for mathematics departments offer potential.

78. The National Strategies are a principal source of training for teachers of mathematics, subject leaders being the main recipients. The termly development meetings aim to update subject leaders and equip them to work with their colleagues on particular foci using a range of materials that are devised nationally and delivered locally. The National Strategies now use more systematic methods of checking on the effectiveness of this model of training and its impact in the classroom.
79. Over the past decade, changes in staffing structures within local authorities have led to a decrease in the number of senior staff with significant responsibility for mathematics. One consequence has been a reduction in opportunities, beyond those provided by the National Strategies, for teachers to participate locally in innovative or developmental work. Some of the larger local authorities, however, are able to support networks of teachers and run annual conferences where good practice in mathematics is shared.
80. A further source of training in secondary schools is provided by awarding bodies in relation to current and new examination courses. At present, however, external assessments place too little emphasis on assessing pupils' depth of understanding of concepts and application to substantial problems. It remains to be seen whether the introduction of functional mathematics and the possible second GCSE in mathematics will have had a positive effect on teaching.
81. Over the past few years, school-based training days have increasingly concentrated on whole-school issues such as assessment for learning and pupils' behaviour and attendance. Schools do not make enough use of this time for subject-specific development. Meetings of secondary departments and primary school staff in some schools provide opportunities for professional development which are most effective when tailored to their particular needs.
82. The new professional standards and arrangements for performance management provide schools with a framework to support collaboration between staff and the sharing of good practice. While collaboration offers potential for professional development, the potential will not be realised if teachers' development needs have not been identified accurately enough.
83. Overall, opportunities for professional development are fragmented and not matched closely to teachers' individual needs. They do not help them to identify what they need to do to improve their subject expertise and how they might do it, building this up systematically. The most urgent needs are to develop primary and non-specialist teachers' subject knowledge and secondary teachers' subject-specific pedagogy. In

particular, many teachers might benefit from professional development on planning and teaching for understanding.

84. One of the recommendations of the Smith Report was to establish a National Centre for Excellence in the Teaching of Mathematics to provide strategic leadership for and coordination of continuing professional development in mathematics, with regional centres to support local communities and delivery.¹² The National Centre has just completed its second year. Through its web portal, teachers have access to a wide range of information. Recent developments include tools for self-assessment of subject knowledge but, at present, these are not linked to training or distance learning modules. Pilot materials to support the development of subject leadership are being trialled and there are plans to develop these into accredited courses.
85. A prime reason for improving professional development is the need for schools to nurture and develop their staff. This is especially important in secondary schools, many of which experience severe difficulties in recruiting teachers and departmental leaders. Some schools are 'growing their own' mathematics staff, through a combination of further study of mathematics and classroom practice.
86. Current developments in the 14–19 mathematics curriculum, such as the introduction of specialised diplomas, which include a functional mathematics component, are likely to increase the pressure on the supply of teachers of mathematics.¹³ A range of national initiatives is attracting more people of diverse backgrounds and experience into teaching mathematics, including graduates whose studies included a more limited amount of mathematics. Participants on mathematics enhancement courses, which are undertaken before trainees start teacher education courses, were excited about learning more mathematics and keen to start teaching it. In primary schools, the programme Every Child Counts will require the specialised training of a large number of staff. These developments provide additional weight to the argument for good expertise in mathematics to ensure learners make secure progress.

¹² *Making mathematics count: The report of Professor Adrian Smith's inquiry into post-14 mathematics education*, The Stationery Office (937764), 2004. www.mathsinquiry.org.uk/report/index.html

¹³ Annex C shows the QCA's timeline for changes in the secondary mathematics curriculum and associated assessments.

Part B: Every child's mind should matter in mathematics

87. This section of the report explores issues which are central to improving pupils' understanding, enjoyment and achievement in mathematics. The different sections examine different aspects of teaching and learning in mathematics, but they are inextricably related. Good practice in one area is generally informed by strengths in another. Similarly, weaknesses or shortcomings can have a pervasive effect. At the centre is the teacher of mathematics. What he or she does enables or impedes pupils' progress in mathematics and the degree to which they become mathematically self-confident and equipped for the future.

Tests and examinations: what is the score?

Do improving results tell the whole story?

88. Test and examination results provide a mixed picture overall, with some cause for celebration but also for concern. Inspections, especially of secondary mathematics, show evidence of significant problems and that apparent improvements are insecure. In particular, the rising trends in attainment are not generally being matched by identifiable improvements in pupils' understanding of mathematics or in the quality of teaching. Instead, the evidence suggests that much is due to the increased level of intervention with underachieving pupils and those on key borderlines of performance, coupled with teaching that focuses on the skills required by examination questions and extensive use of revision.
89. This trend can also be seen, albeit to a lesser extent, in primary schools. The proportion of lessons in which pupils made good or better progress was substantially higher in primary than in secondary schools. However, as pupils approach Year 6, intervention, 'booster' and revision classes increase in effort to optimise pupils' performance in the national tests. These, and teaching that focuses on the tests, often have a narrowing effect on pupils' experiences of mathematics in Year 6, at the expense of strengthening their understanding of underpinning concepts.
90. In many secondary schools, the progress pupils make on a day-to-day basis in mathematics lessons does not, on its own, account for their longer term achievement as measured by national tests and examinations. This sometimes significant gap is being closed through the positive impact of factors outside lessons which include many pupils' notable efforts in revision and homework clubs, and schools' systems to track pupils' performance and then intervene to bolster it.
91. Achievement and standards in 'using and applying mathematics' remain lower than in other areas of mathematics. These higher order skills underpin what it means to behave mathematically. It is of serious concern, therefore, that national tests do not require pupils to use and apply mathematics in substantial tasks through which they are able to decide what approaches to adopt, use a range of mathematical techniques in exploring the problem, find solutions, generalise and communicate

their reasoning. The importance of these skills is highlighted in the new National Curriculum's key processes and they underpin the recently published standards for functional mathematics.^{14, 15} However, unless external assessments reflect these important processes, they are unlikely to influence a significant shift in teaching and learning mathematics.

92. National Key Stage 2 tests assess elements of 'using and applying mathematics' through short real-life problems expressed in words so that pupils have to decide what calculation they need to do. In many schools, having identified this as an area of weakness, a focus on solving such problems has led to some improvement. However, too often, this narrow approach is pupils' only experience of 'using and applying mathematics'. They rarely have the opportunity to investigate open-ended problems. Moreover, few schools have a secure grasp, backed up by formal records, of pupils' progress and attainment in this aspect.
93. The picture is bleaker in secondary schools. Teachers seldom plan explicitly for 'using and applying mathematics' and it is very rare for schools to assess this aspect of pupils' learning separately. This is a statutory part of the Key Stage 3 teacher assessment but there are no national arrangements to check whether or how well this is being done or to gather information about what such data might show. Furthermore, the removal of coursework as a component of GCSE mathematics from 2009 means that teachers will no longer routinely assess 'using and applying mathematics' at Key Stage 4 either. As with primary schools, inspection evidence confirms that pupils have little experience of applying their mathematics to a variety of open-ended, novel or complex tasks and, without such opportunities to investigate and extend their reasoning skills, standards in this crucial aspect remain lower than other areas of the mathematics curriculum.

Equipping pupils for the future

94. A clear message of this report is that, in most schools, mathematics does not contribute sufficiently to the five outcomes of the Every Child Matters agenda. Too few schools take seriously their duty to teach pupils to use and apply mathematics for themselves, an important skill in promoting their economic well-being and interpreting information to help them be healthy and stay safe. A small number of the schools surveyed illustrated what is possible but 'using and applying mathematics' was an area of relative weakness in the majority of schools.
95. Too many secondary pupils expect to find learning mathematics difficult and seem to accept that this is so. They know the difference between being proficient at carrying out techniques and understanding the underlying mathematical ideas. They recognise that they often learn methods by following teachers' illustrative examples and working through many exercises, obtaining correct answers without really understanding why. Some pupils quite like the security of being given rules and

¹⁴ The new secondary *National Curriculum in mathematics* can be found at <http://curriculum.qca.org.uk/key-stages-3-and-4/subjects/mathematics>

¹⁵ The *functional skills standards: mathematics* (Ref QCA/07/3166), QCA, 2007.

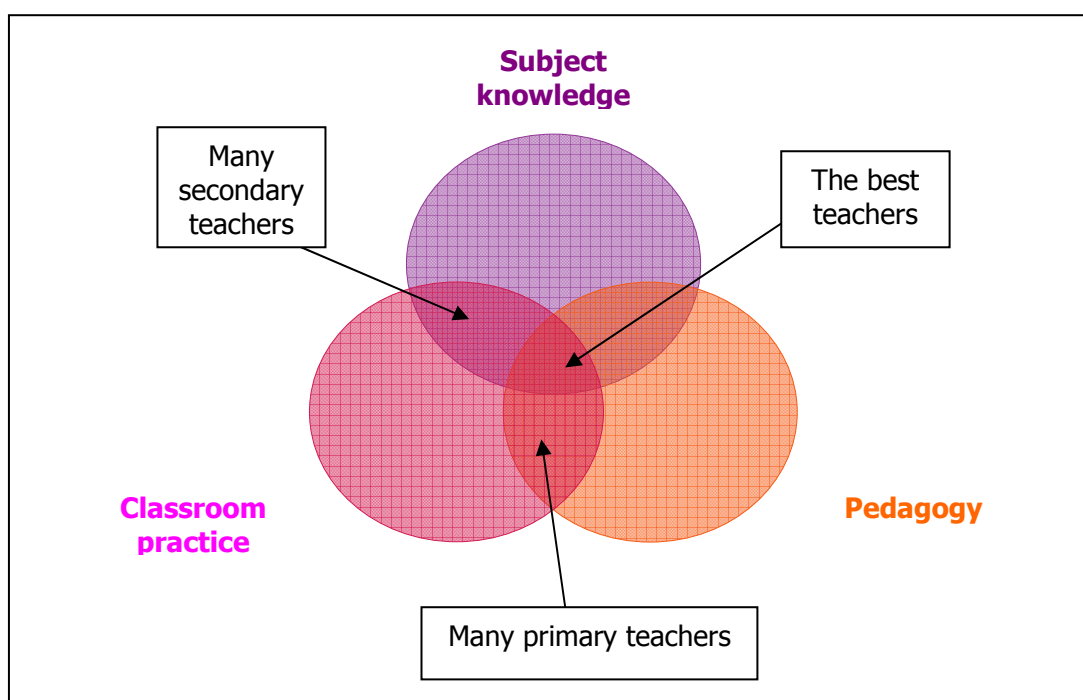
structured methods, but tend to become dependent on them and, in turn, on their teachers. Many pupils refer frequently to prompts provided by the teacher about how to carry out a technique, but such methods, memorised without understanding, often later become confused or forgotten, and subsequent learning becomes insecure. Moreover, such an approach fragments the mathematics curriculum.

<p>Weaker factors: right answers but insecure learning</p>	<p>A Year 8 lesson in which pupils learnt a method for solving simple equations of the form $2x + 5 = 13$ and $5x - 7 = 8$ but with superficial understanding. Although the technique was initially demonstrated correctly, pupils' thinking was not developed in a way that would support further learning.</p>
<p>The teacher demonstrated correctly the technique of adding to or subtracting from each side of the equation to create a simpler equation, such as $2x = 8$ and $5x = 15$, and then dividing by the coefficient of x. Pupils were set an exercise with around 20 similar questions. The teacher gave help as needed until most had answered several questions. The answers were read out and pupils gave themselves a mark out of 20, with many scoring full marks.</p> <p>Noticing that every question had the same format, and that several pupils had omitted their working, the inspector tried out some variations with a few pupils. These pupils tackled $3 + 18x = 42$ with confidence. When asked to explain how they arrived at their (incorrect) answer of $x = 8$, they said they had subtracted 18 and divided by 3. Their choices were based on the position of the numbers 3 and 18 in the equation, and not their meaning.</p> <p>By setting all questions in the same format, pupils took a short cut to the answers, and did not think about the method they had originally been taught. Critically, the teacher gained a false impression of pupils' learning, believing they could now solve simple equations, whereas this was in fact restricted to a particular subset of such equations. Pupils could not extend their approach to any other equations.</p>	
<p>How might it be improved?</p>	<p>To improve learning in this lesson the teacher, when first demonstrating the method, could have checked that pupils understood each step by selecting examples in which the positions of the numbers within the equations varied. Following this by independent work that included a range of equations would allow any misconceptions to be exposed. Insisting on good presentation of solutions would help reinforce the need for logical thinking.</p>

96. The vast majority of pupils of all ages are capable of more. Even those with little experience of solving problems showed that they were able to do so, with coaxing through a mix of encouragement and prompting, when inspectors presented them with challenging problems. They were willing to engage in discussion, although many struggled to use appropriate mathematical language to explore problems and express their ideas. A surprising finding was that younger pupils, rather than the older and higher attaining, were often more willing to 'have a go', suggesting that the experience of learning mathematics and passing examinations is not equipping pupils with confidence in their mathematical ability.

Teachers' subject knowledge, pedagogic skills and classroom practice

97. The best teachers combine deep knowledge and understanding of the subject with well informed appreciation of how pupils learn mathematics. They are committed to exploiting both to ensure that every learner makes the best progress possible in sustainable ways. This report refers to this combination of subject knowledge and pedagogy as subject expertise. The third component is experience of classroom practice. The Venn diagram (below) illustrates the three components. Ideally, all teachers of mathematics would be located in the central section: they would have appropriately deep and broad subject knowledge, a good understanding of how pupils learn mathematics, and teaching that is underpinned by both.



98. Subject expertise is highly significant in teaching and learning. Key characteristics of good subject expertise include:

- understanding the conceptual difficulties of different topics
- being aware of the mathematical progression of ideas
- striking a good balance between developing skills, knowledge and understanding
- having a sense of how deeply to cover topics with different groups of pupils
- understanding the value of good recall and high levels of competence in basic techniques
- knowing which topics need to be marked diagnostically
- being able to understand and evaluate pupils' suggestions and individual methods and answer their reasonable questions

- knowing what questions to ask to probe understanding and to identify and tackle pupils' misconceptions
- giving pupils responsibility for their own understanding by making them aware of what they are learning and by helping them to think mathematically so they can make sense of it
- informally assessing pupils as they work and adapting the lesson accordingly
- using appropriate vocabulary and correct mathematical notation, and maintaining mathematical correctness
- being able to make links between different areas of mathematics
- knowing how to use visual representations and practical resources to enhance understanding
- selecting a rich variety of examples, exercises, practical activities, problems and extended investigations that challenge and extend pupils' understanding
- understanding the role of 'big ideas' in mathematics, such as the number line, place value, multiplicative reasoning, and inverse processes
- knowing some of the history of mathematics and its applications.

Good subject expertise in practice

99. Many pupils meet mathematical ideas one at a time and therefore do not appreciate the links within mathematics at all levels. Pupils who can shade in $\frac{3}{4}$ of a shape often have difficulty placing $\frac{3}{4}$ on the number line; they do not think of it as a number. When solving simultaneous linear equations algebraically, many pupils do not realise that the solution they have found corresponds to the coordinates of the point of intersection of two straight lines. Good teaching ensures that these important connections are forged, but the most effective teachers enable the pupils to make the links for themselves.

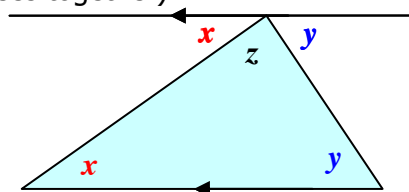
Prime practice: making links within mathematics	<p>A sixth-form further mathematics lesson in which students investigated properties of 2x2 matrices of the form:</p> $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$
<p>Guided by the teacher, the students found that each matrix represented an enlargement of scale factor $r = \sqrt{a^2 + b^2}$, with rotation by θ about the origin, where $r \cos \theta = a$, $r \sin \theta = b$. They established that the matrices had the same properties as complex numbers of form $a + ib$, and that the set formed a group and a ring. They therefore found links across the three topics of matrices, complex numbers and algebraic structure.</p>	

100. Effective teachers anticipate pupils' likely misconceptions and are skilled in choosing resources and particular examples to expose misconceptions and check that their understanding is secure.

Prime practice: visual aids	Good use of a 10x10 grid with low-attaining Year 6 pupils helped understanding of tenths and hundredths and their fraction and decimal representations.
The teacher made excellent use of 10x10 grids on an interactive whiteboard to identify fractions $\frac{1}{10}$ and $\frac{1}{100}$, seamlessly moving to 0.1 and 0.01 and their equivalences with percentages. Pupils enjoyed using blank grids and the interactive whiteboard, for example when converting $\frac{1}{2}$ to a percentage. They were excited by their success. The teacher also used the whiteboard exceptionally well to dispel misconceptions, for example, when pupils suggested $\frac{8}{10}$ is 8%.	

101. The following example illustrates how good expertise enables mathematical correctness to underpin explanations without making the ideas inaccessible. It pays attention to detail and is precise.

Prime practice: mathematical correctness	A Year 7 lesson on the sum of the angles in a triangle.
<p>The teacher started the lesson by rehearsing what pupils knew about the angle properties of intersecting and parallel lines. Pupils were expected to recall facts about vertically opposite, corresponding, alternate and supplementary angles. Pupils could explain that vertically opposite angles had to be equal because they were both supplementary to the same angle (totalling 180 degrees together).</p> <p>Pupils cut out triangles and tore off the corners, but each pupil had a different triangle, and all were pasted onto a class poster. The teacher elicited from them a proof that the angles of a triangle are supplementary by drawing a line through a vertex parallel to the opposite side of a triangle and encouraging them to apply their existing knowledge.</p>	



102. More usually, the approach adopted to finding the sum of the angles in a triangle is to carry out the practical activity described above and, having stuck the triangle corners into pupils' books, to conclude that 'This shows that the angles of a triangle always add up to a straight line. Therefore a triangle has 180 degrees'. Pupils often then successfully complete an exercise on missing angles in triangles. However, they do not appreciate that more than this demonstration was needed to be convincing about the sum of the angles of a triangle.

103. Many pupils do not grasp that angles relate to measures of turn. Instead, they see them as a space that occupies the corner of a shape. Examples of good teaching that helped to develop pupils' conceptual understanding included opportunities for all pupils to estimate angles using dynamic geometry software that showed a rotating arm and helped pupils to distinguish angle measure from that of lengths. When measuring angles for themselves, the diagrams were presented with angles in different orientations, some to be measured clockwise and some anticlockwise.

104. Teachers who have effective subject expertise know how to structure learning in ways that allow pupils to connect apparently different topics, and build on their earlier learning.

Prime practice: the mathematical progression of ideas	A Year 7 lesson introducing the calculation of probabilities.
A teacher, realising that probability is a difficult idea for many pupils, had made sure that pupils were used to marking fractions and decimals on a number line before they met the idea of the probability scale. He emphasised the need to consider equally likely outcomes in calculating probabilities through groups of three pupils playing a game which was based on the number of heads obtained from spinning two coins. At each turn, the player whose number came up scored a point. The pupils quickly learnt that this game was 'not fair'. They realised that there were four equally likely outcomes (tail-tail, tail-head, head-tail, head-head) rather than three (0, 1, 2 heads) and that this was why 1 head was more likely than either 0 or 2 heads.	

More typically, pupils complete questions on calculating probabilities, for example 'There are five red and three white balls in a bag. What is the probability of obtaining a red ball?', but do not connect this to work on marking probability estimates on a number line.

105. At some stage, most teachers are asked questions by pupils about the usefulness of what is being taught. Many feel uncomfortable with these, especially with more abstract concepts, often resorting to answering, 'It's on the syllabus'. Few talk about specific applications or explain the power of being able to think mathematically. Up-to-date and easily accessible advice on careers in the field of mathematics is limited.

Prime practice: applications of mathematics	A teacher's response to 'Why do we have to learn algebra? What use will it be?'
The teacher reminded the pupils that algebra is important in science because formulae are needed to express the laws of science; spreadsheets use algebraic formulas and are a very powerful tool used by thousands of businesses; and computer graphics require complicated algebraic methods to make sure that objects are portrayed correctly. He also pointed out the power of algebraic notation as a means of communicating within mathematics.	

106. The range of pupils' errors and misconceptions when they learn algebra means that their written work on algebraic topics is an important source of clues to their thinking. The best teachers focus on pupils' errors as a learning point. They spot the significant misconceptions which are illuminated by pupils' mistakes. Skilful teachers select a range of questions for the pupils to tackle, making sure that all pupils are challenged and each is exposed to potential misconceptions. Pupils' work on 'collecting like terms' frequently shows many to be fluent when only positive terms are involved, but that negative terms often cause errors. In many classes, especially

where large numbers of questions have been set, pupils mark work themselves and the opportunity to diagnose errors is missed. Sometimes the low proportion of harder questions, or their positioning at the end of an exercise where few pupils reach them, results in pupils obtaining a high proportion of correct answers which can give teachers a misleading picture of their understanding.

Prime practice: the need for diagnostic marking	A teacher's views on the importance of marking pupils' work on 'collecting like terms' in algebra.
In the lesson, the teacher developed an activity for pupils adapted from a National Strategy training pack. She provided plenty of graduated practice that gradually introduced complications such as negative terms. Her explanation to the pupils emphasised that terms could be added and subtracted in any order, provided that 'positive terms stay positive and negatives stay negative'. She explained that many pupils had trouble with this topic, and that she used a specific range of questions to enable the different types of error to be revealed. Because they would need this skill often in later work, she would mark the work herself to identify any misconceptions.	

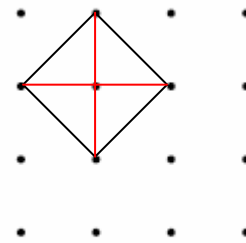
Teachers' subject expertise

107. Teachers vary in their expertise. Often, one aspect is much better developed or more evident than another. This unevenness varies largely, but not exclusively, by phase. In general, secondary mathematics teachers are specialists who have good subject knowledge and many want to share it with their pupils. Primary teachers, who are not usually subject specialists, often have strong pedagogic skills; they focus on pupils as individual learners.
108. The majority of secondary mathematics teachers are well qualified, although less strongly than in science and English. The recently published *Secondary School Curriculum and Staffing Survey* shows that nearly a half have degrees in mathematics or allied subjects such as engineering, and a further quarter have degrees in mathematics education or postgraduate teaching qualifications.¹⁶ While they are equipped to teach effectively, many do not exploit their depth of subject knowledge in their teaching. A significant minority of secondary teachers of mathematics are not specialists. The survey found that around one in six of all mathematics lessons for pupils in Years 7 to 13 are taught by staff who have no post A-level qualification in mathematics or mathematics education; this represents an increase since 2002. Some of these teachers are specialists in subjects other than mathematics or hold senior leadership responsibilities and spend only a small part of each week teaching. Other non-specialists and some unqualified staff teach mathematics full time.

¹⁶ *Secondary School Curriculum and Staffing Survey 2007*: Research report DCSF-RR026, NFER, 2008. www.dfes.gov.uk/research/programmeofresearch/projectinformation.cfm?projectId=14979&keyword=staffing%20survey%20&keywordlist1=0&keywordlist2=0&keywordlist3=0&andor=or&type=5&resultspage=1

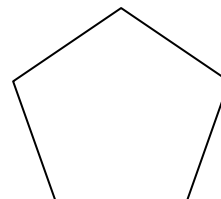
109. The vast majority of primary teachers have little knowledge or experience of mathematics beyond courses they studied at school, such as GCSE or O-level mathematics. There are limited opportunities on one-year postgraduate teacher education courses for trainees to enhance and deepen their knowledge of mathematics substantially. Consequently, it is often their own learning from much earlier that informs their view of mathematics. While primary teachers become familiar with the mathematics they are teaching, often to the same age group in consecutive years, the bigger picture of progression within the subject and the interrelationships between different aspects or topics can be lost, or sometimes never properly understood.

<p>Weaker factors: knowledge of geometry</p>	<p>Primary teachers justifying whether a shape is a square or not.</p>
<p>As part of a professional development activity, groups of primary teachers drew various squares on dotted grids. In justifying whether the shape illustrated on the right was definitely a square, many offered approaches such as fitting corners of sheets of paper into the corners of the shape but this does not justify the angles being exactly 90°. When prompted, they could explain why the four sides were of equal length. Further coaxing led to identification of pairs of angles of 45°, and hence right angles.</p> <p>The teachers did not recognise the succinct proof gained by spotting that the diagonals were of equal length and bisected each other at right angles. Many did not recall this property of squares. Yet high-attaining pupils in Key Stage 2 are expected to use the properties of quadrilaterals to classify four-sided shapes. The mathematical knowledge and understanding of many teachers do not equip them to do this effectively.</p>	



110. In primary and secondary schools, teachers sometimes make mistakes in their explanations or when demonstrating solutions. Such moments can trigger fruitful discussion and debate. However, when errors reflect teachers' weak understanding of mathematics and are not noticed or corrected, pupils can be left confused and in danger of repeating the error themselves. The security of their subsequent learning is also threatened. Limited subject knowledge restricts the dialogue teachers can have with pupils and the range of questions they can ask to probe pupils' understanding. As a result, they are less able to identify misconceptions and move pupils' learning on.

Weaker factors: gaps in subject knowledge	A Year 6 lesson on interior angles of polygons in which a teacher's weak subject knowledge led to pupils' incorrect understanding.
<p>A Year 6 class was investigating the interior angles of regular polygons. Many found this difficult, but higher-attaining pupils had found that a pentagon has interior angles of 108°. The teacher said that this was not correct and encouraged them to divide 360 by 5 to get the answer, stating 'the angles in any polygon add up to 360'. This gave the answer of 72°, which puzzled the most able pupils as the interior angles were clearly bigger than right angles. Other pupils appeared to just accept the rule which they then incorrectly applied to other polygons.</p>	
How might it be improved?	The teacher had not realised that this was a gap in her knowledge. Possibly, she had confused previous knowledge about external angles which do sum to 360°. If she had had the confidence to ask the able pupils to explain their answer, she might have recognised her error. She returned to the pupils' answer of 108° in the next day's lesson.



111. Secondary teachers can also have gaps in subject knowledge and these can have negative consequences for their pupils. One of the problems about teachers' subject knowledge is that they 'don't know what they don't know'. For example, when practising past GCSE papers, pupils in a high-attaining Year 11 set struggled to solve the equation:

$$\frac{2}{2x - 3} + \frac{1}{x - 1} = \frac{2}{x}$$

They sought help from the teacher who proceeded to give them an incorrect solution. This confused a few of the pupils, but most thought he was right. He had inverted the equation wrongly, thus:

$$\frac{2x - 3}{2} + \frac{x - 1}{1} = \frac{x}{2}$$

112. Most primary teachers have good pedagogic skills, many of which they are able to transfer from one curriculum area to another. This is also true of some non-specialist secondary teachers. Giving attention to what pupils say and matching work to their individual needs are the main reasons why mathematics teaching is generally better in primary than secondary schools.
113. In discussion with inspectors, although most secondary teachers recognised the importance of pedagogic skills in mathematics, they often commented on the pressures of external assessments on them and their pupils. Feeling constrained by these pressures and by time, many concentrated on approaches they believed prepared pupils for tests and examinations, in effect, 'teaching to the test'. This practice is widespread and is a significant barrier to improvement.

Weaker factors: poor use of subject expertise	<p>A Year 8 lesson following homework on 'collecting like terms' in algebra. Although the teacher realised that the pupils had had difficulty with the homework, the teacher's subsequent approach was unhelpful because it was not mathematically precise and compounded existing misconceptions.</p>
<p>In the starter activity, pupils took turns to go to the interactive whiteboard to match equivalent algebraic expressions by collecting like terms. The examples involved positive terms only. Pupils then marked their homework on the same topic, the teacher reading out answers. When it became apparent that several had not completed the homework, the teacher amended his lesson to explain the topic again, using the imagery of counting apples, bananas, and so on. When one question involved both c (cats) and c^2, the teacher stretched the imagery, saying 'c^2 is different to c. It is like a cat with two black ears'. Despite the bizarre imagery, pupils were eventually able to complete the homework. However, the idea that algebraic terms represent objects is unhelpful; indeed such a method reinforces this misconception.</p>	
How might it be improved?	<p>The teacher might have found it useful to have marked this homework himself.</p> <p>An approach to collecting like terms that generalises arithmetic would be more powerful mathematically; for instance two 7s added to three 7s makes five 7s might help with $2c + 3c = 5c$, and 7^2 is clearly different from 7.</p>

114. To raise standards further in ways which can be sustained requires investment in developing subject expertise, particularly primary and non-specialist teachers' subject knowledge and secondary teachers' pedagogical skills. Some very recent initiatives are potentially useful starting points. The challenge will be to create an environment where professional development is valued and reaches all teachers of mathematics.

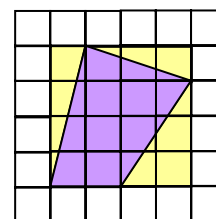
Assessment for understanding: the teacher as detective

Why is assessment in lessons important?

115. Teachers who assess well show a fundamentally different approach from those who do not. They focus on ensuring that all pupils move on from their differing starting points. This is immediately apparent in the way they actively seek assessment clues throughout the lesson, adapting their approach in response to the learning needs of individuals or groups.
116. In the best lessons during the survey, the teachers were perceptive listeners and observers, both in interpreting pupils' responses to questions and when moving among pupils who were working on tasks and exercises. Their strong subject expertise enabled them to monitor and intervene in a timely way. They strove to understand how each pupil was thinking and were concentrated on using this as a basis for structuring learning rather than aiming to convey a particular mathematical method. It was their focus on trying to interpret what was in pupils' minds, to help them make better sense of the mathematics for themselves, that singled out these lessons. They realised that, unless they knew how a pupil was thinking, they would not be in a position to help them learn effectively.

117. These teachers' lessons were well designed. Conceptual approaches and practical activities promoted understanding, allowing common misconceptions to surface and be tackled constructively. The teachers gave pupils opportunities to think for themselves, to explain their reasoning and to apply what they knew creatively. Listening to, observing and marking pupils' responses to these rich prompts provided the teachers with useful evidence from which they diagnosed difficulties and the need for further challenge.

Prime practice: building understanding	Conceptual approaches to the teaching of area meant that Year 5 pupils could do much more than find the area of a rectangle using a formula.
<p>A primary teacher emphasised that the area of a shape was measured by the number of 1cm squares it could hold. By drawing rectangles to the correct size on squared paper, she had helped pupils to give meaning to the numerical answers. They had initially counted squares. She checked carefully that pupils had recognised the rows and columns of squares in their rectangles and could use them to calculate the area of a rectangle more quickly. She introduced triangles and many other shapes through geo-boards. Pupils devised their own strategies for composite shapes, including halving to get triangles, and discussed them with other pupils.</p>	



118. Circulating quickly around the class allowed teachers to check on all pupils and intervene where appropriate. For example, they checked what pupils were writing on their mini whiteboards; they saw where pupils were leaving gaps in quick mental tests or were finding them too easy; after giving a one-minute task, they listened to what each pair of pupils was discussing; having asked pupils to draw axes or a diagram, they confirmed rapidly that all had done so correctly and were ready for the next step. In these lessons, teachers did not allow themselves to spend too long responding to a few pupils who asked for help, thereby leaving others unnoticed. They soon assessed if some pupils were stuck or lacking in confidence and adapted the lesson to make sure everyone could attempt the work.
119. Conversely, in far too many lessons, teachers did not build on the clues to pupils' levels of understanding provided by their responses. The two principal reasons for this relate to subject knowledge and pedagogy. First, teachers may not have appreciated the significance of the clues in helping them to move every pupil forward during the lesson and, second, many are reluctant to seek and use pupils' responses flexibly, modifying their teaching to capitalise on what pupils know and think.
120. Commonly, teachers remain at the front of the classroom during starter activities, while introducing a topic and during class discussions. This means they miss important information about the questions pupils find difficult or too easy and do not recognise where an early slip is interfering with pupils' learning. For example, many pupils draw axes with unequal spaces between units which prevent them from plotting straight line graphs correctly, yet they continue to work on them unnoticed for too long. In some classrooms, because of the way the desks were arranged or because primary pupils were seated closely together on the carpet in front of the

teacher, it was difficult for teachers to move quickly to where they could see each pupil's work.

121. Many secondary teachers spent more time circulating while pupils were working individually or in groups but this was often in response to pupils who held up their hands. Many others were unvisited and, consequently, the teacher was insufficiently informed about their learning.
122. Sometimes, teachers did not assess the extent of pupils' difficulty accurately. Typically, the teacher asked a question, very few hands went up, a selected pupil answered it well, and the teacher assumed that all the class knew and understood. Actually, the pupils' books and discussions indicated that many were unclear. Such lessons might have been more effective if the teacher had circulated to check on pupils' books and discussions, posing questions to verify whether particular pupils who were likely to have difficulty did, in fact, understand. Focused questions to particular pupils, the use of mini whiteboards and discussion with individuals were ways this was achieved in the more successful lessons.

Weaker factors: teacher not circulating	A lesson starter in which the teacher was unaware that pupils' progress was very variable.
<p>A low-attaining Year 7 class was given a worksheet as a quick lesson starter. It contained several questions of the form $400 + 300 = 600 + \dots$</p> <p>The teacher did not circulate to check anyone's work so did not realise that some pupils had written 1,300 and attempted the remaining questions incorrectly as additions. While some pupils finished very quickly, others had managed only a few questions. The speed of responses showed that the pupils who already knew how to do this work were not extended and those who did not know gained little benefit.</p>	
How might it be improved?	<p>If the teacher had moved around the class quickly checking pupils' first answers, or used mini whiteboards for the starter activity, he would have identified those pupils who were making the mistake of adding the three numbers. Continuing to circulate as pupils worked would show the teacher who was struggling and who was not challenged by the task.</p> <p>Learning might have been better if the questions had been tailored to pupils' prior attainment, perhaps through two or three worksheets at different levels of challenge.</p>

123. The usual structure to primary lessons means that teachers do not often move around the class during the main part of the lesson when pupils are working in groups. This is because a couple of the groups are the planned focus of attention of the teacher and the teaching assistant, while the other pupils are expected to work independently. However, at this stage of the lesson their progress is in danger of stalling or slowing, either because they get stuck or because the work is undemanding.

124. Sometimes, a pupil is heard to say to the teacher, 'When I am stuck and ask you a question, you don't tell me the answer. You just ask me another question!' This is often a sign of a skilful teacher, whose questions are helping to get to the bottom of what the pupil is thinking and hence what the difficulty is. Carefully constructed questions can enable the pupil to resolve the difficulty themselves. For example, a pupil gave the answer of 3.6 when working out 0.6×0.6 in her head. The teacher asked her what the value of 6×0.6 was. Realising that this answer was 3.6, the pupil herself then corrected her initial answer to 0.36. The same is true during whole-class interaction where listening to responses and asking follow-up questions help to build pupils' understanding.
125. For a variety of reasons, teachers find it easier to assess pupils' knowledge and skills rather than their understanding. Sometimes this relates to their own expertise. For example, many teachers do not teach in a way that promotes understanding. The view that grouping pupils by ability ('sets') removes the need to match teaching to their different needs is common. Consequently, when teachers pay too little attention to individual pupils' needs, it is not always clear to inspectors whether this stems from weaknesses in teachers' assessment skills or from their view that an approach matched to different needs is unnecessary. In this respect, teachers in secondary schools are not helped when their department has only one scheme of work for all pupils in a year group, despite the wide differences in their needs.

Improving the use of assessment in lessons

126. Many schools in the survey had 'assessment for learning' as a major part of their school improvement plans. This often included a focus on improving the quality of teachers' questioning. Sometimes, this led to identifying useful 'key questions' in teachers' planning which helped teachers and pupils to focus on and review the important learning points in a lesson. Teachers said they found it very beneficial to discuss with each other what, in fact, the key questions were.
127. The use of lesson objectives, often linked to particular GCSE grades or National Curriculum levels, was another focus in many schools. Various methods were used to match the intended learning to the needs of the different groups in a class but the tasks were not consistently well designed to enable the planned learning to take place or matched to pupils' previous learning. In many lessons in secondary schools, all the pupils tackled the same work: in effect, teachers were expecting some pupils' learning to be more successful than others.
128. Marking was a common theme of whole-school training. In the best examples seen during the survey, careful thought had been given to interpreting policies on marking and feedback in terms of teaching and assessing mathematics. The most successful approaches often included pupils assessing their own and each other's work. More frequently, however, teachers were expected to implement whole-school policies. Some schools required marking which consisted only of comments. Others expected every piece of work to be assigned a National Curriculum level, sub-level or GCSE grade. Many teachers found these policies difficult to apply in mathematics when pieces of work met one element of a particular level description only in part. In any case, the level descriptions in mathematics are linked closely to specific content and

often are not directly related to descriptions at the lower levels. One approach to 'comment only' policies was to mark diagnostically, focusing on topics where misconceptions were common. The effectiveness of this approach depended on the quality of the tasks. In some cases, the work teachers set and the questions they asked were not challenging or probing enough and therefore did not stimulate useful assessment.

Pupils' self-assessment

129. The use of self-assessment, a crucial part of pupils taking responsibility for their own learning, is improving slowly. It is more advanced in primary schools but often still in its early stages. Good practice during the survey included meaningful reference to the learning objectives during the lesson as well as thoughtful use of checklists and regular assessments to aid pupils' understanding of their progress and attainment. But much of pupils' involvement in self-assessment was relatively superficial: pupils showed their understanding and confidence through systems such as 'traffic lights' and 'thumbs up' or 'thumbs down' but, because pupils wanted to succeed and were eager to please, some signalled their understanding too readily when using such systems. Sometimes they confused 'understanding' with knowing how to carry out the steps of a taught method independently. This blurred the usefulness of their self-assessment.
130. Self-assessment includes opportunities for pupils to mark their own work. Wise teachers used checking procedures to apply some quality control, realising that pupils needed guidance if they were to be reliable judges of their understanding. They also made sure that they marked certain topics directly. When answers were simply read out for pupils to check, usually towards the end of the lesson, some pupils responded by altering their wrong answers so that it appeared that they had no errors. This did not help them to learn. Moreover, some teachers did not realise that this was happening. Managers need to challenge such superficial attention to pupils' learning, and teachers' role in it, through closer monitoring.

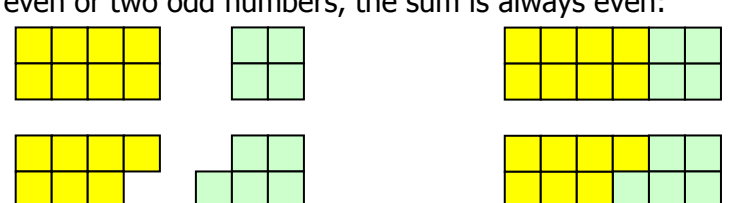
Using and applying mathematics: pupils as mathematicians

131. The best practice had 'using and applying mathematics' at the heart of teaching and learning in mathematics: pupils were viewed as budding mathematicians and developing their understanding was of paramount importance. This was reflected in a shared ethos, pervading the teaching, learning and curriculum, and focused on approaches that developed pupils' understanding and their independence in using and applying mathematics. Such practice was relatively rare, although, in some schools, reflection had led to a deliberate drive towards improving pupils' understanding of mathematics – an encouraging sign.
132. Good curricular planning provided pupils with opportunities to apply mathematics to a variety of interesting tasks, enabling them to choose approaches, reason and refine their thinking in the light of their solutions. Teachers encouraged pupils to discuss mathematical problems in depth and this helped to build their confidence. In a primary school where developing pupils' understanding was promoted effectively,

pupils were confident in 'thinking aloud' and were not afraid to have their mistakes used to help others.

Prime practice: discussion	An interesting approach to ratio and proportion with Year 6 pupils with lots of discussion.
<p>The teacher engaged pupils throughout the lesson by incorporating many activities and encouraging discussion and argument in pairs until an answer was agreed. A reverse approach to solving problems was effective in getting pupils to think about clarity of expression. The teacher put one cup of fruit juice and two cups of water in a jug and one cup of fruit juice and three cups of water into another jug. The contents of both jugs were poured into a bowl, which, by then, contained 2,800ml of the mixture.</p> <p>The teacher posed the question: how many millilitres of fruit juice are in the bowl? Pupils worked in pairs with jottings on mini whiteboards. Many struggled at first, argued with each other, but eventually worked out that $\frac{2}{7}$ of the mixture would be juice. Pupils were then asked to write a question, in words not just numbers, to match the problem they had just solved. As the lesson went on, middle-attaining pupils in the group completed more, similar questions and higher-attaining pupils were given some requiring much deeper thinking.</p>	

133. Where the curriculum was not planned in this way, the unspoken assumption was that pupils would acquire these important skills incidentally, for example through tackling occasional investigative tasks or solving a range of short questions set in context, such as calculating the volume of a child's toy formed from a cone and a hemisphere, or working out the cost of taking a group of pupils to the theatre. While it is important that pupils can solve such problems, they provide limited opportunity for independent thought or for making generalisations, a crucial element of behaving mathematically.
134. Over the last two years, many primary schools have increased their emphasis on solving word problems in mathematics. Schools sometimes identified pupils' weak literacy skills as an impediment. However, instead of helping pupils to think their way through the problems, some schools tried to give them a 'recipe' for solving them. This did not help to develop pupils' skills of independent reasoning or communication.
135. A key area for improvement in primary schools was to develop more open-ended tasks that provided opportunities for pupils to investigate mathematically, for example, choosing how to solve a more complex task, deciding what to work out, how to present answers, and whether tackling the task in a different way might give an alternative answer. Other activities might encourage pupils to make or explore general statements. For example, pupils might investigate the truth of the statement: 'Adding two odd numbers together always makes an even number.' Problem solving was an integral part of the best mathematics lessons. Pupils enjoyed trying out different methods: 'We like learning for ourselves.' In an atypical lesson, pupils discussed excitedly with one another whether an exact answer to $11 \times ? = 100$ could be found.

<p>Weaker factors: pseudo investigation</p>	<p>The way tasks are framed can close down opportunities for pupils to investigate mathematics. In this example, Year 5/6 pupils were nominally 'investigating' what happens when different combinations of odd and even numbers are subtracted. They had previously found rules for adding.</p>
<p>The teacher had presented the task as one of identifying 'the correct rule' by asking: 'Does odd minus odd give an odd or even answer?' Confident that a rule existed, pupils simply tried one example and inferred general rules from single examples.</p> <p>The teacher's approach meant that pupils never engaged with the possibility that there might be no consistent rule. In the previous lesson they had been guided to record three rules for addition ($O+O=E$, $O+E=O$, $E+E=E$) but reasons why the rules worked and links between the rules were not made clear.</p>	
<p>How might it be improved?</p>	<p>The teacher's questions could have been phrased in an open way: 'What happens when you add or subtract two odd numbers?' followed later by: 'Does this always happen?'</p> <p>Learning would have been better if the teacher had given the pupils greater independence by not assuming that a rule had to exist and by providing practical equipment such as interlocking cubes so that they could represent odd and even numbers visually. Pupils could then illustrate their explanations and justify rules. They could also have been encouraged to look for unifying ideas, for instance when adding two even or two odd numbers, the sum is always even:</p> <div style="text-align: center;">  </div> <p>The teacher might have benefited from guidance on teaching approaches for such tasks and about what aspects of using and applying mathematics pupils could develop through the activity.</p>

136. Inspectors explored general statements, real-life problems, or both, in discussing mathematics with pupils. Many of the pupils had an underdeveloped ability to use and apply mathematics because they had not been given enough opportunities. They seemed to enjoy the discussions and, occasionally, also asked mathematical questions of the inspectors. Even when pupils were completely unfamiliar with discussing concepts such as generalisation and proof, inspectors found that they were almost always willing to engage in dialogue. With coaxing, they expressed their ideas, but often struggled to use appropriate mathematical language. Older pupils tended to think in silence, responding only when they had reached a final solution, whereas younger pupils tried out ideas and happily discussed them with each other. This reflects secondary pupils' lack of experience of discussing mathematics regularly in lessons. Simple strategies, such as pupils sharing a mini whiteboard and jointly giving answers, increase the amount of mathematical talk in a lesson. This also helps pupils to be less anxious about making mistakes because the responsibility is shared.
137. The lack of development of 'using and applying mathematics' is a prime reason why pupils' understanding of mathematics lags behind their proficiency in executing

techniques and recalling facts. Some primary schools and secondary departments, often well led and reflective on their practice, were introducing approaches that focused more on pupils' learning; for example, starting lessons with tasks or problems that made pupils think. The teachers encouraged discussion and debate, enabling pupils to learn for themselves and from each other. Teachers' enthusiasm was a key contributory factor. However, in nearly half the schools in the survey, provision for 'using and applying mathematics' was inadequate or barely adequate. It was generally weaker in secondary schools than in primary. In both, teachers need support and guidance in planning, teaching and assessing 'using and applying mathematics' and, thereby, in teaching for understanding.

<p>Prime practice: teaching mathematical thinking</p>	<p>The context of this Year 9 problem-solving lesson was a series of questions about the number of permutations of letters in different names, such as LUCY, ALI or WAYNE.</p>
<p>Rather than show pupils the standard formula, the teacher provided them with an opportunity to find their own solutions. This was not as haphazard as it might seem, because he also had a very clear idea about which kinds of thinking he wanted to encourage and the point he wanted pupils to move towards. This type of problem solving might be characterised as 'open in the middle' rather than open-ended.</p> <p>The lesson objectives were: 'Pupils will learn: the value of working systematically to solve problems; to refine their understanding of the methods they develop; to refine their oral and written explanations of their methods; and the value of reducing a problem to a simpler case.'</p> <p>For much of the lesson, the teacher's role was to listen to pupils explaining their ideas, to encourage and nurture any systematic thinking, and to intervene with additional problems when appropriate. Mini-plenaries were used as appropriate to encourage pupils to share their ideas with the class, draw out key ideas that emerged and stimulate further thought about variations on the original problem. By the end of the lesson, most pupils had worked out that the number of permutations of n distinct letters would be $n! = 1 \times 2 \times 3 \times \dots \times n$. More importantly, they understood the importance of making systematic lists and therefore understood in a concrete sense the recursive nature of the solution: that a five-letter word could begin with any of the five letters, followed by any of the 24 permutations of the other four letters, giving $5 \times 24 = 120$, and that 24 arose as 4 (starting letters) \times 6 (ways of arranging the other three letters), and 6 as 3×2, and so on.</p> <p>Variations of the problem were held in reserve, such as EMMA, ANN, GEMMA and DONALD, leading to the generalised problem of counting permutations when some letters repeat. Many pupils recognised that having two letters the same halved the number of possibilities and that having three letters the same reduced the number further, but realised that this needed more thought.</p>	

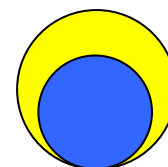
Pupils' enjoyment and views of mathematics

138. The majority of primary pupils enjoyed learning mathematics, commenting, 'I like the teachers. They make lessons fun', and seeing its importance: 'You can use it in real life.' As with secondary pupils, they liked lessons that were interactive, varied with practical and other interesting activities and which used ICT. Year 6 pupils, however, often said they had little hands-on experience of ICT in comparison with previous

years at their school, citing the need to prepare for tests as the reason. Pupils of all ages said they liked being able to work in groups and pairs. In many of the secondary schools, however, these enjoyable features did not occur regularly in mathematics lessons. Pupils' experience of them varied widely within the same school, between teachers, and from one key stage to another.

139. Most secondary pupils were relatively ambivalent about mathematics. Typical comments included, 'It's good when you understand it' and 'It's boring. I prefer active or creative subjects, like dance and art.' Most said they 'quite enjoy it' but few secondary pupils cited mathematics as their favourite subject, even those who were doing very well. Most expressed feelings of satisfaction when they were able to reach correct answers on a particular topic and lack of enjoyment when they could not. Their enjoyment therefore varied, often hinging on how well they could 'do' a particular topic.
140. A remarkable degree of consistency existed in much of what pupils said about their experience of learning mathematics, no matter what kind of school they attended, from village primary to large inner-city comprehensive. There was strong agreement on the features of mathematics lessons that they enjoyed: 'It's fun working in groups' and 'Working with someone else helps you understand, especially if they ask you questions.' They disliked too much talk by the teacher and other pupils' misbehaviour. Many pupils, especially in secondary schools, described a lack of variety, which they found dull. Typically, their lessons concentrated on the acquisition of skills, solution of routine exercises and preparation for tests and examinations. 'Every lesson, you have to answer questions from the textbook. It gets boring.' They contrasted this with occasional lessons they enjoyed where they did investigations, tackled puzzles, sometimes working in groups, and used ICT independently. Often such lessons happened at the end of term and were regarded as end-of-term activities rather than being 'real maths'.
141. It should be of serious concern nationally that so many secondary pupils seemed to accept that this was what learning mathematics should be like, despite their recognition that teaching and learning in other subjects were not the same. Many pupils, including some in Year 6, said they expected to find mathematics difficult and that they would have to practise and memorise the methods they were shown if they were to succeed in tests and examinations. They were prepared to do this because they knew success in mathematics was important. One Year 2 pupil said, 'I'm not looking forward to being in Year 6. They have to work really hard for the tests.'
142. When asked, most pupils recognised the difference between just getting answers right and understanding the work. Nevertheless, many of those observed in lessons were content to have the right answers in their books when they did not know how to arrive at them. They frequently replicated steps in a method without thinking and sometimes altered answers, or waited until the teacher read them out before writing them down. This view that mathematics is about having correct written answers rather than about being able to do the work independently, or understand the method, is holding back pupils' progress.

<p>Weaker factors: doing well but without understanding</p>	<p>A pupil correctly calculated the areas of circles of radius 5cm and 7cm, by applying the standard formula $A = \pi r^2$.</p>
<p>When the inspector asked her whether it was reasonable that the second area was nearly twice as much as the first, she immediately assumed her answer must be wrong, as she was not used to being asked to interpret her answers. After further discussion, it became clear that she had learnt how to use the formula to calculate the area of a circle as a number, but could not say what was meant by the area of a circle. The few circles drawn in her book were all the same size. She had learnt a method to obtain answers to a problem she did not understand.</p>	
<p>How might it be improved?</p>	<p>The pupil's understanding would have been better if the teacher had:</p> <ul style="list-style-type: none"> established at the beginning how well each pupil understood the concept of area provided experience of finding the areas of shapes drawn to their actual size used pupils' previous knowledge about areas of shapes to approximate the area of a circle, for example by sandwiching it between squares and/or polygons. <p>Understanding would also have been strengthened if the pupil had been asked the sorts of questions that would have made her think about what she was learning and how to interpret results.</p>



143. Most pupils considered that their work was generally at about the right level of difficulty, and said they enjoyed it when it was challenging. However, their views on and experiences of problem solving were more mixed. Some preferred the comfort of routine exercises where they knew when they had got the right answers, but older pupils, and more able pupils of all ages, tended to relish the challenge of solving problems; one sixth-form student said, 'I like the satisfaction of getting something right eventually which I may not have understood at first.' Pupils would like more that added interest and relevance to their learning by relating to everyday situations. Some secondary pupils, particularly the more able, said they enjoyed the challenge of algebra.
144. Mathematics at advanced level is commonly perceived to be more difficult than most other subjects. Many higher-attaining Year 11 pupils indicated some anxiety about their ability to cope with A-level mathematics. Choosing to study it related mostly to its usefulness in supporting subjects they wanted to read at university. Considerable variation remains in the advice schools provide for potential AS and A-level students. Some schools discourage all but the highest-attaining pupils; others allow wider access but then often do not adapt teaching approaches and support the students well enough, so failure and drop-out rates during the course are high. The pathways towards qualifications that pupils follow after taking GCSE early are not always thought through carefully enough.

145. Many pupils, particularly secondary pupils, spoke of finding work difficult in lessons and needing to ask for extra help afterwards. 'I don't like maths because I'm no good at it. But the help from teachers is good.' This need reveals the insecurity of their learning in lessons. The pupils praised their teachers' dedication in providing this support, as well as revision for tests and examinations. Many pupils indicated that they were more open with each other (and inspectors) about difficulties in understanding work in lessons than they were with their teachers. This reflects pupils' perceptions of the importance of right answers in mathematics: being stuck is viewed negatively rather than as a challenge to be overcome.
146. There is no reason why pupils should not both enjoy and understand mathematics: they are not contradictory. Many pupils lack confidence in their own abilities and worry about getting answers wrong, even when their methods are right. They can find the more open-ended and unusual problems daunting. Nevertheless, short-term satisfaction gained from a page of ticks for correct answers to repetitive exercises is fragile if the mathematics itself is not understood well enough to be applied independently to unusual problems. Subsequent learning is then built on conceptual sand.
147. Some schools have begun to collect pupils' views of their experience of learning mathematics. This is a positive move, and action to respond to them has the potential to make learning mathematics more fun for pupils and their teachers. A pupil said: 'Maths makes you think – your mind grows intellectually. But sometimes you learn more from your friends than your teacher. Explaining builds up our confidence.'

Conclusion

148. This report is published at a time of considerable change in mathematics, four years on from the Smith report and shortly after the publication of the findings of the Williams review. New curricula and new qualifications are emphasising the need for pupils to be mathematically functional. This is exactly the time to get teaching and learning right in mathematics: it is what current and future generations of children and young people need if they are to be properly mathematically equipped.
149. Teaching and learning, the curriculum, and leadership and management of mathematics are all stronger in primary schools than in secondary schools. Many secondary schools face significant challenges in finding good teachers of mathematics. However, continuous improvement is hard to achieve and the sharp rise in the national test results for primary schools, which began a decade ago, has now more or less reached a plateau. Many primary teachers still need better knowledge of mathematics if they are to be enabled both to help the lowest attaining pupils reach the expected standards and to challenge the highest attainers. In secondary schools, despite the attention given to GCSE mathematics, many pupils are not doing as well as they should. Pupils' learning is based too much on their acquisition of methods, rules and facts, as part of the strong focus on tests and examinations, and too little on their understanding of the underpinning concepts, on connections with their earlier learning and other topics in mathematics, and on

helping them to make sense of mathematics so that they can use it independently. Teaching that concentrates on how pupils learn mathematics effectively is what is needed most in secondary schools.

150. While there are many good teachers of mathematics, the teaching of many others is ordinary and, often, narrowly focused, particularly in secondary schools. Some simple things that teachers could do would lead to quick improvements, such as making the most of pupils' oral responses and clues to their thinking shown in their written work. However, other issues are deeper seated and will require more sustained action from teachers, subject leaders, senior staff, all those who work with teachers, and those responsible for national policy and assessment. This report aims to stimulate and support such a drive for improvement.

Notes

This report is based on evidence from inspections of mathematics between April 2005 and December 2007 in a range of maintained schools in England. The sample of 192 schools was selected to form a cross-section of schools geographically and by institutional type, including middle schools, voluntary-aided schools and specialist mathematics and computing colleges. No school judged inadequate in its last whole-school inspection was included in the sample.

The sample included six primary schools and one secondary school selected specifically on the basis of high achievement or good practice known from previous inspection. To allow for fairer comparison between primary and secondary schools, the judgements made on the quality of provision in these schools have not been incorporated into the percentages quoted in the report.

The report also draws on evidence from the first two years of section 5 whole-school inspections from September 2005 and from inspection visits relating to the evaluation of the National Strategies.

Further sources of evidence include the annual reports of Her Majesty's Chief Inspector for the three years from 2004 to 2007 and other reports published by Ofsted including *Evaluating mathematics provision for 14–19-year-olds*.¹⁷ The evidence was also informed by discussions with those involved in mathematics education, including teachers and pupils, subject leaders and senior staff in schools, academics, policy makers and others within the wider mathematics community.

Evidence from the findings of this survey has contributed to the review by Sir Peter Williams into mathematics teaching in early years settings and primary schools.¹⁸

¹⁷ *Evaluating mathematics provision for 14–19-year-olds* (HMI 2611), Ofsted, 2006; www.ofsted.gov.uk/publications/2611

¹⁸ *Independent review of mathematics teaching in early years settings and primary schools*, 2008; <http://www.standards.dcsf.gov.uk/primary/mathematicsreview>

Further information

Advisory Committee on Mathematics Education (ACME)

www.acme-uk.org/

ACME is an independent committee which acts as a single voice for the mathematical community, seeking to improve the quality of education in schools and colleges. It advises Government on issues such as the curriculum, assessment, and the supply and training of mathematics teachers. Its most recent conference was on 'Mathematics in STEM: a policy perspective'. It has published various position papers, including *Mathematics and level 3 diplomas* (February 2008) and *The future of primary mathematics* (May 2006).

Department for Children, Schools and Families (DCSF)

www.dcsf.gov.uk

The department's website provides links to many aspects of mathematics education, including the work of the National Strategies (links below).

The report from the Government's inquiry into post-14 mathematics education *Making mathematics count: The report of Professor Adrian Smith's inquiry into post-14 mathematics education*, 2004, can be found at www.mathsinquiry.org.uk/report/index.html

Department for Innovation, Universities and Skills (DIUS)

www.dius.gov.uk/

The department's website provides links to adult basic skills learning, for example on numeracy at <http://geton.direct.gov.uk/> or information for parents at www.direct.gov.uk/en/Parents/Schoolslearninganddevelopment/HelpingYourChildToLearn/DG_4016596

Further Mathematics Network

www.fmnetwork.org.uk

The Further Mathematics Network provides support for teachers and students of advanced level mathematics and further mathematics, providing tuition in further mathematics for those students who would benefit from studying it but would not otherwise have the opportunity to do so.

Mathematical Careers

www.mathscareers.org.uk

This recently established website provides information for young people of all ages, from Key Stage 3 to graduate level, who are interested in finding out about careers and opportunities that an education in mathematics can present. It covers a range of queries

and careers including mathematics, statistics, engineering, medicine, finance, computer graphics and forensic science. It also contains information for teachers, parents, careers advisers and employers.

National Association of Mathematics Advisers (NAMA)

www.nama.org.uk

Membership of NAMA is open to advisers, inspectors, consultants, and providers of advice, inspection and guidance within the field of mathematics education. The association is dedicated to promoting high-quality mathematical education in the United Kingdom.

National Centre for Excellence in the Teaching of Mathematics (NCETM)

www.ncetm.org.uk

The NCETM was launched in June 2006. It is responsible for enhancing professional development across mathematics teaching in all settings and with learners of every age, and promotes collaboration between teachers. The web portal is the gateway to the breadth of the centre's national activity and to each of the nine sub-regions. A wide range of information and links are provided, for example to online courses, self-evaluation tools, support for subject leaders, publications and details of forthcoming events. Materials relating to a recent conference on 'The potential of ICT in mathematics teaching and learning' are at

www.ncetm.org.uk/Default.aspx?page=13&module=res&mode=100&resid=9006

National Strategies

www.standards.dfes.gov.uk/secondary

www.standards.dcsf.gov.uk/primary

The Primary and Secondary National Strategies for school improvement are part of the Government's programme for raising educational standards in line with the Every Child Matters agenda.

Information about the Key Stage 4 intervention programme 'Study Plus' can be found at

www.standards.dfes.gov.uk/intervention/home.html

Materials for assessing pupils' progress at Key Stage 2 are at

www.standards.dcsf.gov.uk/primaryframework/assessment/app/

Qualifications and Curriculum Authority (QCA)

www.qca.org.uk

The role of the QCA is to maintain and develop the National Curriculum and associated assessments, tests and examinations. The curriculum section of the QCA's website includes details of the National Curriculum and schemes of work, and is found at

www.qca.org.uk/qca_104.aspx. The new secondary mathematics curriculum is at <http://curriculum.qca.org.uk/key-stages-3-and-4/subjects/mathematics>

Office of the Qualifications and Examinations Regulator (Ofqual)

www.ofqual.gov.uk

Ofqual began its interim work as regulator for qualifications, examinations and tests in England on 8 April 2008. Until legislation is passed by parliament, it will operate as part of QCA.

Learning and Skills Improvement Service (LSIS)

LSIS, a new sector-led improvement organisation, was formed in April 2008 to bring together the work of two agencies, the Centre for Excellence in Leadership (CEL) and the Quality Improvement Agency for Lifelong Learning (QIA). It will focus on learners and support the development of leadership and excellence in the further education and skills sector. Information about CEL and QIA may be found at www.centreforexcellence.org.uk and www.qia.org.uk

Royal Society

www.royalsociety.org.uk

The Royal Society, the national academy of science of the UK and the Commonwealth, established ACME in 2002 with support from the Joint Mathematical Council and funding from the Gatsby Foundation. The Royal Society's recently published report, *The UK's science and mathematics teaching force*, anticipates a future shortfall of 33% in the supply of mathematics teachers.

Subject associations

www.teachernet.gov.uk/professionaldevelopment/professionalassociations/subjectassociations/

There are many subject associations in mathematics, some of which are listed on the teachernet website. These include the Association of Teachers of Mathematics (ATM) www.atm.org.uk and The Mathematical Association (MA) <http://www.m-a.org.uk>

Training and Development Agency for Schools (TDA)

www.tda.gov.uk

The TDA is the national agency responsible for the training and development of the school workforce.

United Kingdom Mathematics Trust (UKMT)

www.ukmt.org.uk/

This registered charity organises mathematics challenges and enrichment activities for schools and colleges.

Annexes

Annex A. Schools visited for this survey

Primary schools	Local authority
All Saints Benhilton CofE Primary School	Sutton
Allfarthing Primary School	Wandsworth
Birchen Coppice First School	Worcester
Bishops Lydeard Church of England Voluntary Controlled Primary School	Somerset
Blackdale Middle School	Norfolk
Blessed Robert Widmerpool Catholic Primary & Nursery School	Nottingham
Branton St Wilfrid's Church of England Primary School	Doncaster
Burbage Junior School	Leicestershire
Cam Everlands Primary School	Gloucestershire
Canon Maggs CofE Junior School	Warwickshire
Carrington Primary and Nursery School	Nottingham
Caynham CofE Primary School	Shropshire
Chaulden Infants' and Nursery	Hertfordshire
Collierley Primary School	Durham
Copeland Road Primary School	Durham
Cranford Junior School	Hounslow
Crownfield Infant School	Havering
Days Lane Primary School	Bexley
Dedham Church of England Voluntary Controlled Primary School	Essex
Eastrop Infant School	Swindon
Exford Church of England First School	Somerset
Ferncumbe CofE Primary School	Warwickshire
The Green Way Primary School	Kingston upon Hull
Finstock Church of England Primary School	Oxfordshire
Fleetwood Chaucer Community Primary School	Lancashire
Grange Junior School	Swindon
Greenhill Primary School	Oldham
Greenside Primary School and Children's Centre	Tameside
Grendon Underwood Combined School	Buckinghamshire
Hey-with-Zion Primary School	Oldham

Hillview Primary School	Halton
Holy Family Catholic Primary School	Knowsley
Holywell Primary School	Hertfordshire
Jessie Younghusband School	West Sussex
John T Rice Infant and Nursery School	Nottinghamshire
Katesgrove Primary School	Reading
Keldmarsh Primary School	East Riding of Yorkshire
Kelsey Primary School	Lincolnshire
Kender Primary	Lewisham
Langworthy Road Primary	Salford
Leys Farm Junior School	North Lincolnshire
Little Heath Primary	Coventry
Longvernal Primary School	Bath & North East Somerset
Lunsford Primary School	Kent
Mark Cross Church of England Aided Primary School	East Sussex
Mayflower Primary School	Tower Hamlets
Muschamp Primary School and Language Opportunity Base	Sutton
Newbold and Tredington CofE Primary	Warwickshire
Nightingale Infant School	Derby
Northdown Primary School	Kent
Offley Infant School	Cheshire
Our Lady Star of the Sea Catholic Primary School	Lancashire
Parklands Primary School	Northamptonshire
Pirton Hill Infant School	Luton
Potley Hill Primary School	Hampshire
Reigate Primary School	Derby
Robert Mellors Primary and Nursery School	Nottinghamshire
Sir John Offley CofE (VC) Primary School	Staffordshire
St Ambrose Barlow Catholic Primary School	Wigan
St Anne's Catholic Primary School	Hampshire
St Benedict's Catholic Primary	Warrington
St Boniface RC Primary School	Wandsworth
St George's, Bickley Church of England Primary School	Bromley
St Gregory's RC Primary	South Tyneside

St James's Hatcham Church of England Primary School	Lewisham
St Joseph's Catholic Primary School	Gloucestershire
St Joseph's Catholic Primary School	Harrow
St Mark's Church of England Primary School, Eccles	Kent
St Mary's CofE Primary School, Moston	Manchester
St Mary's RC Primary School	Salford
St Mary's RC Primary School	Oldham
St Michael's CofE Primary School	Devon
St Saviour's Catholic Infant School	Cheshire
St Thomas of Canterbury RC Primary School	Medway
Staining Church of England Voluntary Controlled Primary School	Lancashire
The St Michael's Church of England Primary School, Thorpe on the Hill	Lincolnshire
Thornton Dale C of E (VC) Primary School	North Yorkshire
Thorpe Hesley Junior School	Rotherham
Victoria House Pupil Referral Unit	Croydon
Willand School	Devon
Wingham Primary School	Kent
Wrightington Mossy Lea Primary School	Lancashire
Wykeham Primary School	Havering
Wyton on the Hill Community Primary School	Cambridgeshire
Secondary schools	Local authority
All Saints' Catholic High School	Sheffield
Allendale Middle School	Northumberland
Altrincham Grammar School for Boys	Trafford
Archbishop Blanch CofE VA High School, A Technology College and Training School	Liverpool
Archbishop Holgate's School	York
Archbishop Ilsley Catholic School	Birmingham
Arrow Vale Community High School – a Specialist Sports College	Worcestershire
Bexleyheath School	Bexley
Birkenshaw Middle School	Kirklees
Blenheim High School	Surrey
Boldon School	South Tyneside
Bourne Grammar School	Lincolnshire

Bournville School and Sixth Form	Birmingham
Braunton School and Community College	Devon
Bullers Wood School	Bromley
Caedmon School	North Yorkshire
Canon Palmer Catholic School	Redbridge
Carre's Grammar School	Lincolnshire
Castle Manor Business and Enterprise College	Suffolk
Chapter School	Medway
Clapton Girls' Technology College	Hackney
Coundon Court School and Community College	Coventry
Cranford Community College	Hounslow
Crompton House CofE School	Oldham
Dulverton Middle and Community School	Somerset
Durham Johnston Comprehensive School	Durham
Eston Park School	Redcar & Cleveland
Estover Community College	Plymouth
Falinge Park High School	Rochdale
Hardley School and Sixth Form	Hampshire
Harrop Fold School	Salford
Hassenbrook School Specialist Technology College	Thurrock
Havelock School	North East Lincolnshire
Heathfield High School	Leicestershire
Heaton Manor School	Newcastle Upon Tyne
Henbury School	Bristol
Highbury Fields School	Islington
Hinchingbrooke School	Cambridgeshire
Hitchin Boys' School	Hertfordshire
Holyhead School	Birmingham
Horbury School – A Specialist Language College	Wakefield
Horsforth School	Leeds
Kemnal Technology College	Bromley
King Edward VII Science and Sport College	Leicestershire
King James's School	North Yorkshire
Kingsfield School	South Gloucestershire
Leytonstone Business and Enterprise Specialist School	Waltham Forest

Lutterworth High School	Leicestershire
Maria Fidelis Roman Catholic Convent School FCJ	Camden
Mascalls School	Kent
Moseley School A Language College	Birmingham
Nether Stowe High School	Staffordshire
Northolt High School	Ealing
Oaks Park High School	Redbridge
Oldfields Hall Middle School	Staffordshire
Our Lady Queen of Peace Catholic High School and Engineering College	Lancashire
Painsley Catholic College	Staffordshire
Park House School and Sports College	West Berkshire
Plessington Technology College	Wirral
Pudsey Grangefield School	Leeds
Queen Elizabeth High School	Herefordshire
Queen's Park High School	Cheshire
Ratton School	East Sussex
Reddish Vale Technology College	Stockport
Richard Coates Church of England Middle School	Northumberland
Riverside Middle School	Suffolk
Royds School Specialist Language College	Leeds
Sacred Heart High School	Hammersmith & Fulham
Sacred Heart High School	Newcastle Upon Tyne
Sarah Bonnell School	Newham
Seaton Sluice Middle School	Northumberland
Sir William Robertson High School, Welbourn	Lincolnshire
Southmoor Community School, Mathematics and Computing College	Sunderland
St Bede's School	Surrey
St Bernard's Catholic School	Buckinghamshire
St Bernard's High School and Arts College	Southend-on-sea
St Cuthbert's High School	Newcastle Upon Tyne
St Edward's CofE Comprehensive School, Language College and Sixth Form Centre	Havering
St Ives School, a Technology College	Cornwall
St Joseph's RC Voluntary Aided Comprehensive School	South Tyneside

St Thomas More Roman Catholic High School Aided	North Tyneside
Steyning Grammar School	West Sussex
Stocksbridge High School	Sheffield
Sutton Centre Community College	Nottinghamshire
Testbourne Community School	Hampshire
The Aveley School	Thurrock
The Corbet School Technology College	Shropshire
The Holy Family Catholic School	Bradford
The John of Gaunt School	Wiltshire
The King's School Specialising in Mathematics and Computing	Wakefield
The King's School, Grantham	Lincolnshire
The Lacon Childe School	Shropshire
The Lafford High School, Billingham	Lincolnshire
The Lakelands School, Sports and Language College	Shropshire
The Marches School and Technology College	Shropshire
The Northcote School	Wolverhampton
The Sele School	Hertfordshire
The Skegness Grammar School	Lincolnshire
The Sutherland School	Telford & Wrekin
Thomas Keble School	Gloucestershire
Trinity Catholic High School	Redbridge
Walderslade Girls' School	Medway
Wallington County Grammar School	Sutton
Westley Middle School	Suffolk
Weston Favell School	Northamptonshire
Whitechapel Middle School	Kirklees
Wilsthorpe Business and Enterprise College	Derbyshire
Winterton Comprehensive School with Specialist Status in Engineering	North Lincolnshire

Annex B. The age profile of the pupils in the schools surveyed

Primary

Age range	Number of schools
3–7	3
3–9	1
3–11	23
4–7	4
4–9	1
4–11	38
5–7	1
5–11	5
7–11	6
8–12	2
	84

Secondary

Age range	Number of schools
9–13	9
11–14	3
11–16	34
11–18	58
11–19	1
13–18	2
14–18	1
	108

Annex C. Features of satisfactory and good mathematics teaching

The next table relates to mathematics teaching in primary schools, and the following table to secondary. They compare characteristics of good teaching with those of satisfactory teaching. Each list does not define what constitutes good or satisfactory teaching, but shows the difference between good and satisfactory features. Teaching that encompasses most of the good features may well be outstanding. Similarly, the cumulative effect of many weaker features can slow pupils' progress.

	Features of good mathematics teaching (primary)	Features of satisfactory mathematics teaching (primary)
1	Lesson objectives involve understanding and make what is to be learned in the lesson very clear.	Lesson objectives are procedural, such as descriptions of work to be completed, or are general, such as broad topic areas.
2	Teaching features a successful focus on each pupil's learning. Pupils are clear about what they are expected to learn in the lesson and how to show evidence of this.	Teaching features a successful focus on teaching the content of the lesson. Pupils complete correct work and are aware of the lesson objectives but may not understand what they mean or what they need to do to meet them.
3	The lesson forms a clear part of a developmental sequence and pupils recognise links with earlier work, different parts of mathematics or contexts for its use.	The lesson stands alone adequately but links are superficial; for example, links are made with the previous lesson but not in a way that all the pupils understand.

4	<p>Teachers introduce new terms and symbols meaningfully and expect and encourage correct use.</p>	<p>Teachers introduce new terms and symbols accurately and demonstrate correct spelling.</p>
5	<p><u>Whole class teaching/questioning:</u> Pupils spend enough time listening to teachers' exposition and working to develop their understanding, and teachers move them on when appropriate. Teachers and support staff ensure all pupils participate actively in whole-class activity, such as through using mini whiteboards or partner discussions. When offering answers or accounts, the teacher expects pupils to give explanations of their reasoning as well as their methods. Pupils are challenged if their explanations do not reflect their ability.</p>	<p><u>Whole class teaching/questioning:</u> Teachers give effective exposition but pupils' understanding is limited due to time constraints or not extended due to limitations with the task. Questioning and whole-class activities are pitched appropriately but do not involve all pupils' actively; for example, few hands up, questions directed to few pupils, mini whiteboards held up whenever pupils are ready so not all give answers or some copy from others. Questioning is clear and accurate but does not require explanation or reasoning; pupils describe the steps in their method accurately but do not explain why it works.</p>
6	<p><u>Group/individual work:</u> Teachers monitor all pupils' understanding throughout the lesson. They recognise quickly when pupils already understand the work or what their misconception might be. They extend thinking through building on pupils' contributions, questions and misconceptions to aid learning, flexibly adapting to meet needs and confidently departing from plans. The work challenges all pupils as it is informed by teachers' knowledge of pupils' learning; for example, through encouraging pupils capable of doing so to improve their explanations or use more efficient methods. Work requires thinking and reasoning and enables pupils to fully understand objectives. Pupils can explain why a method works and solve again a problem they have solved a few weeks earlier. Non-routine problems, open-ended tasks and investigations are used often by all pupils to develop the broader mathematical skills of problem solving, reasoning and generalising.</p>	<p><u>Group/individual work:</u> Competent questioning but the teacher may miss opportunities to respond to needs; for example, does not build on errors or sticks too closely to plans. Pupils generally complete work correctly but may have made errors or already understand the work so tasks do not fully stretch the high attainers or support the low attainers. Methods are clearly conveyed by teachers and used accurately by pupils; pupils focus on obtaining correct answers rather than enhancing understanding and questions may not be carefully selected. Skills may be short-lived so pupils cannot answer questions which they have completed correctly a few weeks earlier. Typical lessons consist of routine exercises that develop skills and techniques adequately but pupils have few opportunities to develop reasoning, problem solving and investigatory skills, or only the higher attainers are given such opportunities.</p>
7	<p>Pupils develop independence and confidence by recognising when their solutions are correct and persevering to overcome difficulties because they expect to be able to solve problems; the teacher's</p>	<p>Support generally offered to pupils does not develop independence in solving complete problems; for example, answers are given too readily or the problem is broken down so much that pupils do not know why the sequence of</p>

	interventions support them in estimating and checking for themselves.	steps was chosen. Pupils may ask for help at each step and are given directed steps to take rather than interventions that encourage thinking and confidence that they can succeed.
8	Teaching assistants know the pupils well, are well briefed on the concepts and expected misconceptions, and provide support throughout the lesson that enhances thinking and independence.	Teaching assistants facilitate access of all pupils, though may be less active in whole-class work.
9	Teachers (and pupils) have a good grasp of what has been learnt judged against criteria that they understand; this is shown through pupil discussion, reflection, oral or written summaries, and ascertained by the teacher's monitoring throughout the lesson.	Teachers (and pupils) make some accurate assessment of learning; for example, the teacher correctly reflects in a plenary what many pupils have achieved, pupils make an impressionistic assessment of their learning, such as using traffic lights or against a generic lesson objective.
10	Teachers' marking identifies errors and underlying misconceptions and helps pupils to overcome difficulties. For example, by setting clear targets, which pupils take responsibility for following up and seek to understand where they have gone wrong.	Accurate marking by the teacher identifies errors and provides pupils with feedback; important work has been marked by pupils or teacher.
11	Good use of subject knowledge to capitalise on opportunities to extend understanding, such as through links to other subjects, more complex situations or previously learned mathematics.	Any small slips or vagueness in use of subject knowledge do not prevent pupils from making progress.
12	Pupils exude enjoyment and involvement in the lesson. Pupils are confident enough to offer right and wrong comments. Pupils naturally listen to and respond to each other's comments, showing engagement with them.	Pupils enjoy making progress in an ordered environment. Some pupils offer responses to whole-class questions. Pupils listen to the teacher's and pupils' contributions and respond to them when asked to.

Features of good mathematics teaching (secondary)	Features of satisfactory mathematics teaching (secondary)
Meeting needs and addressing misconceptions	
a Teaching features a successful focus on each pupil learning.	Teaching features a successful focus on teaching some content.
b Teachers monitor all pupils' understanding throughout the lesson, recognising quickly when pupils already understand the work or what their misconceptions might be, for example, circulating to check all have started correctly, spot errors and extend thinking.	Pupils generally complete some correct work but the teacher does not recognise when some pupils are stuck, have made errors or already understand the work, for example the teacher moves on too quickly or does not circulate to check so gives answers or methods when pupils have already done the work correctly.
c The teacher listens carefully and interprets pupils' comments correctly, building on pupils' contributions, questions and misconceptions to aid learning, flexibly adapting to meet needs and confidently departing from plans.	The lesson features competent questioning but the teacher is focused more on what has been asked than on the information about understanding that pupils' responses or lack of responses offers; misses opportunities to respond to needs, for example does not build on errors or pupils' comments that they are stuck, and sticks too closely to plans.
d Work challenges higher and lower attainers, as well as middle attainers, because it is informed by teachers' knowledge of pupils' learning; for example through setting different work for different groups, or encouraging pupils capable of doing so to improve their explanations or use more efficient methods	Pupils complete some correct work that extends or consolidates their competence but does not stretch the high attainers or support the low attainers well, for example pupils are given challenging work only if they finish many routine questions quickly or the numbers used in a problem create barriers to the concept for lower attainers.
e The plenary extends learning and meets the needs identified during the lesson.	The plenary draws the lesson to an orderly close
Understanding concepts and explaining reasoning	
f Lesson objectives involve understanding.	Lesson objectives are procedural, such as descriptions of work to be completed, or are general, such as broad topic areas.
g Lesson activities are structured around key concepts and misconceptions, so that carrying out the activities enhances understanding, for example involving pupils in developing suitable methods to solve problems, selecting questions carefully from exercises. Pupils can explain why a method works and solve again a problem they solved a few weeks earlier.	There is a successful focus on developing skills and obtaining correct answers rather than enhancing understanding; such as providing examples which do not illustrate why the method works, or doing questions identical to worked examples, too many of which are similar and are not carefully selected. These skills may be short-lived so pupils cannot answer questions which they have completed correctly a few weeks earlier.

h	Work requires thinking and reasoning and enables pupils to compare approaches.	Methods are clearly conveyed by teachers and used accurately by pupils; pupils rely on referring to examples, formulae or rules rather than understanding or remembering them.
i	Practical, discussion and ICT work enhance understanding, for example using demonstration and mental visualisation of shapes being rotated, with pairs deciding which method gives the correct answer and why.	Practical, discussion and ICT work is motivating and enables pupils to reach correct answers but is superficial and not structured well enough to enhance their understanding, such as unfocused pair work on a book exercise, group tasks where the highest attainer does all the work or free choice of hands-on ICT.
j	Pupils give explanations of their reasoning as well as their methods.	Questioning is clear and accurate but does not require explanation or reasoning; pupils describe the steps in their method accurately but do not explain why it works, for example discussion activities enable pupils to share approaches but do not ensure they explain their reasoning.
k	Pupils spend enough time working to develop their understanding.	Teachers give effective exposition that enables pupils to complete work correctly but restricts the time they have to develop their understanding through their own work, for example teachers talk for too long, pupils spend too long copying examples, notes or questions, or drawing diagrams.
l	Good use of subject knowledge capitalises on opportunities to extend understanding, such as through links to other subjects, more complex situations or more advanced mathematics.	Any small slips or vagueness in use of subject knowledge do not prevent pupils from making progress.
m	Teachers introduce new terms and symbols meaningfully, they expect and encourage correct use; pupils and teachers use mathematical vocabulary and notation fluently.	Teachers introduce new terms and symbols accurately and demonstrate correct spelling.
n	Lesson forms clear part of a developmental sequence and pupils recognise links with earlier work, different parts of mathematics or contexts for its use.	Lesson stands alone adequately but links are superficial, for example pupils know it is lesson two of five on a topic but not how it builds on lesson one. Contexts or applications are mentioned without indicating how the mathematics may be used in a way the pupils can understand.
o	Non-routine problems, open-ended tasks and investigations are used often by all pupils to develop the broader mathematical skills of problem solving, reasoning and generalising.	Typical lessons consist of routine exercises that develop skills and techniques adequately but pupils have few opportunities to develop reasoning, problem solving and investigatory skills, or only the higher attainers are given such opportunities.

Involving all pupils		
p	Pupils exude enjoyment and involvement in the lesson.	Pupils enjoy making progress in an ordered environment.
q	Teachers ensure all pupils participate actively in whole-class activity, such as through using mini whiteboards in ways which involve all, or partner discussions.	Questioning and whole-class activities are pitched appropriately but do not involve all pupils' actively, for example few hands up, questions directed to few pupils, some not attempting written tasks, mini whiteboards held up whenever pupils are ready so not all give answers or some copy from others.
r	Respect is conveyed for pupils' contributions so that many offer right and wrong comments.	Few pupils offer responses to whole-class questions although their work is generally correct.
s	Pupils naturally listen to and respond to each other's comments showing engagement with them.	Pupils listen to the teacher's and pupils' contributions and respond to them when asked to.
Developing independence in learning and assessment		
t	Pupils develop independence by recognising when their solutions are correct and persevering to overcome difficulties because they expect to be able to solve problems; the teacher's interventions support them in estimating and checking for themselves and in raising their confidence; pupils take responsibility for following up teachers' comments on their work and seek to understand where they have gone wrong.	Pupils produce generally correct work through support that does not develop independence in solving complete problems, such as through providing answers too readily or breaking down the problem so much that pupils do not know why the sequence of steps was chosen; for example, pupils do not attempt hard questions and wait for answers to be read out or check them from the answer book, or focus unduly on obtaining correct answers so amend wrong answers unthinkingly when the correct ones are read out, or ask for help at each step and are given directed steps to take rather than interventions that encourage thinking and confidence that they can succeed.
u	Teachers and pupils have a good grasp of what all pupils have learnt judged against criteria that they understand, not necessarily against learning objectives or targets; this is shown through pupil discussion, reflection, oral or written summaries or explanations, and ascertained by the teacher's monitoring throughout the lesson; for example, both teacher and pupil assess whether the pupil can explain why the formula for the area of a rectangle works.	Teachers and pupils make some accurate assessment of learning; for example the teacher correctly reflects in a plenary what many pupils have achieved, pupils make an impressionistic assessment of their learning, such as using traffic lights or against a generic lesson title like 'solving equations'.
v	Teachers' marking identifies errors and underlying misconceptions and helps pupils to overcome difficulties, for	Accurate marking by the teacher provides pupils with feedback; important work has been marked by pupils or teacher.

	example by setting clear targets to which pupils respond and teachers check against.	
w	Pupils are clear about what they are expected to learn in the lesson and how to show evidence of this.	Pupils complete correct work and are aware of the lesson objectives but they are not clear about which ones pertain to them, what they mean, or what they need to do to meet them, for example when objectives are phrased in terms of 'all', 'most' and 'some' pupils without indicating which pupils, when objectives are written down but pupils do not understand their meaning by the end of the lesson when a large quantity of questions are set and pupils do not know how they relate to the objectives or when pupils do not have an attainable target to work towards.
x	Teaching assistants know the pupils well, are well briefed on the concepts and expected misconceptions, and provide support throughout the lesson that enhances thinking and independence.	Teaching assistants facilitate the production of correct work, but may not be active throughout the lesson and may provide support that leads pupils through so many small steps that independence is not encouraged.